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by Robert G. Dorsch, Don J. Wood, and Charlene Lightner

Lewis Research Center
Cleveland, Ohio

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SUMMARY

A digital distributed parameter model for computing the dynamic response of propellant feed systems is formulated. The analytical approach used is an application of the wave-plan method of analyzing unsteady flow. Nonlinear effects are included. The model takes into account locally high compliances at the pump inlet and at the injector dome region. Examples of the calculated transient and steady-state periodic responses of a simple hypothetical propellant feed system to several types of disturbances are presented. Included are flow disturbances originating from longitudinal structural motion, gimbaling, throttling, and combustion-chamber coupling. The analytical method can be employed for analyzing developmental hardware and offers a flexible tool for the calculation of unsteady flow in these systems.

INTRODUCTION

A variety of analytical models have been employed for calculating the low-frequency (0 to 500 cps) dynamic response of rocket propellant feed systems. Early theoretical investigations (e.g., refs. 1 to 3) were primarily concerned with instabilities resulting from the coupling of propellant feed system flow perturbations with combustion chamber pressure oscillations. More recently instabilities arising from propellant flow perturbations driven by longitudinal structural oscillations have received attention (refs. 4 to 6). In this type of instability the propellant flow perturbations result in engine thrust perturbations, which in turn feed back through the structure to close the loop. The authors of references 1 to 6 have employed small perturbation techniques and have linearized system impedances at mean operating conditions. These simplifications are usually valid for the

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purposes of making closed-loop stability predictions where the growth or decay of small (or incipient) sinusoidal pressure and flow perturbations is considered.

Because of the generally nonlinear nature of liquid flow systems, linearized models cannot be used to calculate the amplitudes and wave shapes of large amplitude pressure and flow disturbances of the type that actually occur in propellant feed systems when instabilities are present. Further, a linearized model cannot be used to calculate the system response to throttling, startup, or shutdown. Nonlinear models have been formulated to calculate the dynamic and transient responses of rocket engine and propellant systems. These models usually employ finite-difference techniques with considerable lumping of parameters and usually have limited versatility because they were formulated to solve specific problems.

An analytical investigation was therefore undertaken at the Lewis Research Center to develop a general nonlinear distributed parameter model for calculating unsteady flow in liquid rocket propellant systems. The objective was to develop a model that can be used to calculate both transient and dynamic responses of a liquid rocket propellant feed system to a variety of different types of input disturbances including structural motion. A pulse synthesis method fundamentally similar to a method of characteristics solution (ref. 7) was chosen for the analytical approach. This method, called the wave plan, was developed in the first phase of the investigation and is described in detail in reference 8.

In the wave-plan method of analyzing liquid filled systems the response of system elements or components to incremental pressure pulses is determined. The incremental pressure pulses are associated with incremental flow changes, which can originate in a fluid system in a variety of ways including mechanical motion of the components. The analyses for the response of individual system elements are incorporated into digital computer subroutines. These subroutines are then combined to form digital computer models of specific systems.

This report illustrates how the wave-plan method can be applied to a liquid rocket feed system. Equations for the major feed system components (tank, pump, injector) are derived and programmed as digital computer subroutines. These equations and subroutines along with those of reference 8 are used to analyze a simple monopropellant (or one side of a bipropellant) feed system. Examples of the calculated responses of the hypothetical feed system to several types of disturbances are presented. Included are flow disturbances originating from longitudinal structural oscillations, throttling, gimbaling, and combustion-chamber coupling. In addition, the effect of local cavitation-induced pump inlet compliance on the suction line response is examined. Pump inlet pressure and flow and combustion-chamber pressure perturbation wave shapes are calculated and presented to illustrate the nonlinear nature of the fluid system. Finally, the effect of pump-inlet flow perturbation suppression devices on the feed system response is examined.

ANALYSIS

Feed System Model

A schematic of the idealized system chosen for analysis is shown in figure 1. The primary elements or components of the propellant feed system are as follows:

(1) A propellant tank with provision for longitudinal structural motion input at the tank outlet junction with the feed line. The motion input provides for feed system coupling when there is mechanical motion of the structure due to longitudinal oscillation of the rocket. The upper boundary of the liquid in the tank is assumed to be at constant pressure.

(2) A constant-diameter propellant feed line (suction line) connecting the supply tank and the pump. The motion of the line itself will be ignored in the examples of this report although it is recognized that very small disturbances can be generated because of viscous effects. (For tapered lines or lines with area changes the motion of the line itself must be included.)

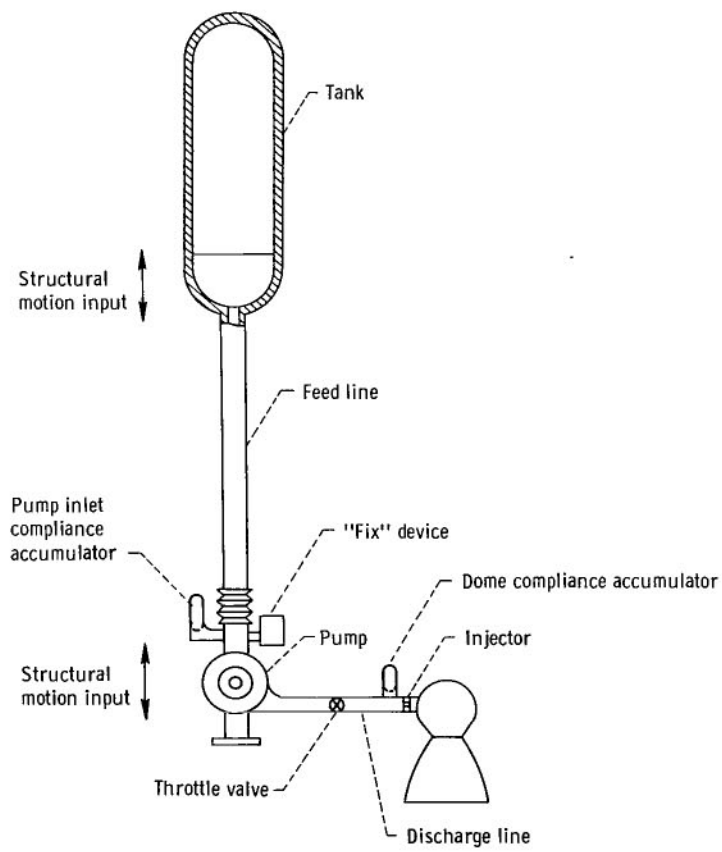


Figure 1. - Schematic of monopropellant feed system.

(3) A pump with provision for longitudinal structural motion input at the junction with the suction line. A device is located at the inlet to the pump in order to simulate the cavitation-induced compliance in the inducer region. This device is represented by an analytical equation.

(4) A fix device (perturbation suppressor) located at the pump inlet station. In the analysis this device can be mathematically combined with or added to the pump cavitation compliance, or it can be treated separately when a mechanical-motion-compensating type of fix device (e. g., constant-volume bellows) is considered (ref. 6).

(5) A pump discharge line. Since the vertical distance from pump to injector is usually small, the discharge line will be assumed to be horizontal.

(6) A nonlinear throttle valve located in the discharge line.

(7) A nonlinear orifice at the termination of the discharge line. This orifice simulates the engine injector. The orifice does not move in the horizontal direction. The compliance of the injector dome area is simulated by a device (represented by an equation) just upstream of the injector. The orifice exit pressure (thrust chamber pressure) is a time-dependent function of the mass discharge of fuel into the thrust chamber. This time dependence includes both a dead time and an exponential delay associated with the gas residence time.

(8) The viscous resistance of the pump suction and discharge lines simulated (when desired) by internal nonlinear (square law) orifices, which approximate turbulent flow friction effects.

Equations describing the response of each of the system elements or components to elastic pressure waves resulting from incremental flow changes must be written. These equations must be written at the discontinuities in the system (e. g., the junction of the suction line with the pump) because the wave-plan analysis supposes a system composed of a discrete number of discontinuities connected by lossless line segments, which serve only to transmit pressure pulses. If any of the elements or components are moving with respect to the primary reference system for the elastic waves (a reference system that moves with the vehicle but does not oscillate), the motion must be taken into account when writing the continuity equations at the discontinuities. In this example specific provision is made for including the effects of mechanical motion at both the tank outlet and at the pump inlet junctions with the feed line.

Component and Junction Equations

Equations for the response of the feed system components to incremental pressure pulses will be derived. The general case in which pressure pulses arrive simultaneously

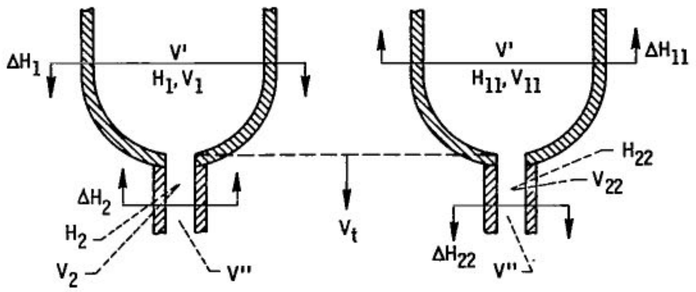


Figure 2. - Conditions at tank outlet before and after wave action.

from both sides of a junction or discontinuity will be considered (ref. 8).

Tank outlet. - Figure 2 shows pressure head and velocity conditions before and after impingement of elastic-wave pressure pulses at a moving tank outlet.

The tank exit is streamlined to minimize viscous losses, for flow going into the feed line, and these losses may be neglected. The energy equation for this case after wave action is

$$\frac{V_{11}^2}{2g} + H_{11} = \frac{V_{22}^2}{2g} + H_{22} \tag{1}$$

(Symbols are defined in appendix A.)

Momentum relations across the wave fronts give the following equations for the pressure head changes associated with the waves:

$$\Delta H_1 = \frac{C_1}{g}(V' - V_1)$$

$$\Delta H_{11} = \frac{C_1}{g}(V' - V_{11})$$

$$\Delta H_2 = \frac{C_2}{g}(V_2 - V'')$$

$$\Delta H_{22} = \frac{C_2}{g}(V_{22} - V'')$$

(See fig. 2 for definition of V' and V''.) Eliminating V' and V'' from these equations gives

$$\Delta H_{11} = \Delta H_1 + \frac{C_1}{g}(V_1 - V_{11}) \tag{2}$$

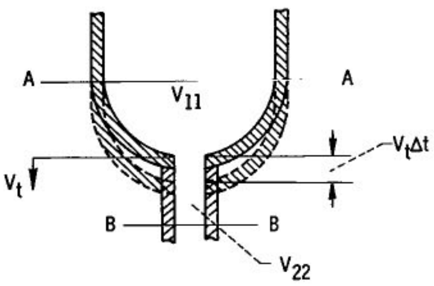


Figure 3. - Continuity conditions at tank outlet.

$$\Delta H_{22} = \Delta H_2 + \frac{C_2}{g}(V_{22} - V_2) \tag{3}$$

The new tank and line pressures after wave action are given by

$$H_{11} = H_1 + \Delta H_1 + \Delta H_{11} \tag{4}$$

$$H_{22} = H_2 + \Delta H_2 + \Delta H_{22} \tag{5}$$

Continuity relations are derived by referring to figure 3, which shows the relative positions of the tank outlet a short time Δt apart. The average velocity of the tank outlet during this time is V_t.

The net volume flow across section AA minus the net flow out of section BB must equal the total storage in time Δt. (Sections AA and BB move with the vehicle but do not oscillate.) Mathematically, the flow difference is given by

$$(V_{11}A_t)\,\Delta t - (V_{22}A_f)\,\Delta t = V_t(A_t - A_f)\,\Delta t$$

Eliminating the incremental time and expressing the result in terms of the ratio of the feed line to tank area R_1 give:

$$V_{11} = V_t(1 - R_1) + V_{22}R_1 \tag{6}$$

The difference in pressure across the tank outlet after the wave action is given by

$$H_{11} - H_{22} = H_1 + \Delta H_1 + \Delta H_{11} - H_2 - \Delta H_2 - \Delta H_{22} \tag{7}$$

Solving equations (1), (2), (3), (6), and (7) simultaneously gives the velocity in the discharge line after the wave action in terms of the initial pressure and velocity conditions, the velocity of the tank outlet, and the magnitude of the impinging waves:

$$a\,V_{22}^2 + b\,V_{22} + c = 0 \tag{8}$$

where:

$$a = \frac{\left(1 - R_1^2\right)}{2g} \qquad b = \frac{C_1 R_1}{g} + \frac{C_2}{g} - V_t \frac{\left(R_1 - R_1^2\right)}{g}$$

$$c = H_2 + 2\Delta H_2 - H_1 - 2\Delta H_1 - \frac{C_2 V_2}{g} - \frac{C_1 V_1}{g} + C_1 V_t \frac{\left(1 - R_1\right)}{g} - V_t^2 \frac{\left(1 - R_1\right)^2}{2g}$$

The positive root of equation (8) gives the propellant velocity in the tank after wave action. The propellant velocity in the feed line is given by equation (6). The magnitude of the generated pressure waves are given by equations (2) and (3), and the tank and line pressures by equations (4) and (5).

Pump. - Because of the present lack of dynamic data for pumps, the relations between net head, suction head, and flow rate for the pump will be assumed to obey the steady-state relations at any given instant. Figure 4 shows plots of typical pump characteristics.

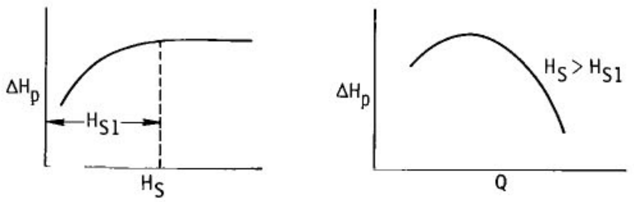


Figure 4. - Typical pump characteristics.

In many cases the relation between the net increase in static pressure head across the pump and the flow rate Q can be accurately described by a quadratic expression:

$$\Delta H_P = A + BQ + CQ^2 \tag{9}$$

To obtain the coefficients, A, B, and C the conventional total head rise against flow curve (fig. 4) is first converted to static pressure head rise against flow corresponding to the particular diameter discharge and suction lines being used. The coefficients, A, B, and C may also be expressed as functions of suction pressure H_S and rotational speed, which makes equation (9) a general expression for static head rise as a function of suction head, flow rate, and rotational speed. For values of suction head greater than H_{S1} (fig. 4) and fixed rotational speed the coefficients A, B, and C remain essentially constant.

For the purpose of determining the effects of pressure wave pulses on a moving pump, the use of coefficients based on the suction head prior to impingement of the waves suffices because only incremental pressure pulses are dealt with in the wave-plan method.

Under most operating conditions a certain amount of cavitation occurs in the inducer region of the pump inlet because of high-speed impeller motion in an area of relatively low pressure. The presence of this vapor in the liquid results in a locally high fluid compliance at the pump inlet. This phenomena is very difficult to express in exact mathematical terms; however, the effect of this lumped compliance must be included in the dy-

namic analyses. This effect will be analytically included by attaching a hypothetical accumulator at the pump inlet.

The compliance a' of the hypothetical accumulator is defined by

$$a' = \frac{d\mathcal{V}}{dH}$$

where $\mathcal{V}$ is the volume of propellant stored in the accumulator.

The accumulator compliance a' is constant only for a linear accumulator; however, it is relatively simple to program a' as a function of line pressure for known nonlinear behavior such as adiabatic or isothermal compression of a gas. If a' is assumed to remain constant and equal to its initial value over an incremental time period, small changes in volume of propellant stored in the accumulator can be related to small changes in pressure head as shown in the following equation:

$$\Delta\mathcal{V} = \Delta H(a')_{H=H_1} \tag{10}$$

where H_1 is the pressure head at the accumulator at the beginning of the interval.

In order to properly simulate the cavitation-induced compliance at the pump inlet by the use of equation (10), the functional form of the pressure-volume relation for a volume of liquid containing vapor bubbles must be known. This relation, however, is not precisely known. It would be expected to be nonlinear, and a hysteresis effect may be present because of a small nonuniform time lag related to heat transfer within the mixture during phase change. For some of the examples in this report the pressure-volume curve for the fluid at the pump inlet will be approximated by assuming it to be hyperbolic in form and to have no time lag. This function is identical in form to the relation for isothermal gas compression in an accumulator.

The compliance for an isothermal accumulator is given by

$$a' = \frac{d\mathcal{V}}{dH} = \frac{H_o\mathcal{V}_o}{H^2}$$

where H_o and $\mathcal{V}_o$ are the initial pressure and volume, respectively, of the gas in the accumulator. It should be pointed out that for a given compliance simulation (value of a') $\mathcal{V}_o$ of the accumulator gas is considerably larger than the total volume of vapor cavities

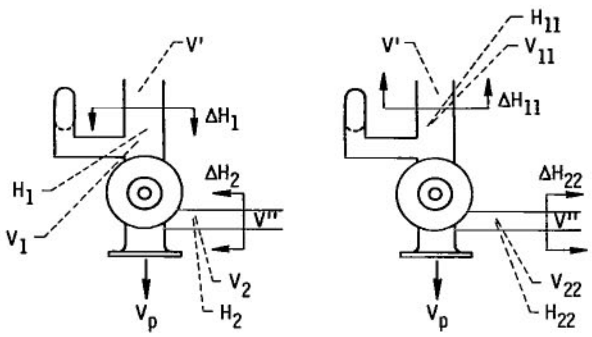


Figure 5. - Conditions at pump before and after wave action.

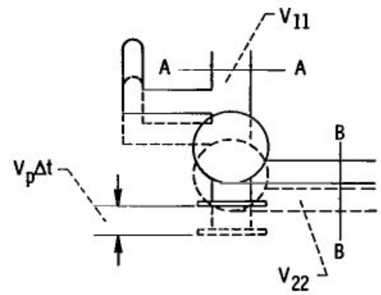


Figure 6. - Continuity conditions at pump.

that would actually exist in the pump inlet region at pressure head H_o, because the phase change from vapor to liquid (or vice versa) associated with a change in head is an important contribution to the net incremental volume change of the fluid.

Figure 5 shows conditions before and after wave action at a pump that is moving parallel to the feed line with average velocity V_p. Conditions are assumed to change at the instant the impinging waves leave the pump and then to remain unchanged for a time Δt. At that time new waves may impinge, and conditions may be once more changed.

The continuity equation for the time period Δt expresses the condition that the flow past boundary AA (fig. 6) equals the storage in the accumulator plus the flow out of the pump (past boundary BB):

$$(V_{11} - V_p)A_f \,\Delta t - \Delta \mathcal{V} = V_{22}A_d \,\Delta t$$

Introducing the relation for the accumulator (eq. (10)) and rearranging terms give

$$V_{11} - V_p - \frac{a'}{\Delta t A_f}(H_{11} - H_1) = V_{22}\frac{A_d}{A_f} \tag{11}$$

The momentum equations are identical with the ones derived for the tank outlet (eqs. (2) and (3)).

The head discharge-flow-rate relation for the pump after wave action is

$$H_{22} - H_{11} = H_2 - H_1 + \Delta H_2 + \Delta H_{22} - \Delta H_1 - \Delta H_{11} = A + BQ + CQ^2 \tag{12}$$

The pump flow rate Q is measured with respect to the pump.

Solving equations (2), (3), (10), (11), and (12) simultaneously for the discharge after wave action in terms of known conditions gives:

$$CQ^2 + \left(B - \frac{C_2}{gA_d} - \frac{C_1}{gA_f\gamma}\right)Q + A + \beta - \alpha - \frac{C_1\Gamma}{\gamma g} = 0 \tag{13}$$

where

$$\alpha = H_2 + 2\ \Delta H_2 - \frac{C_2V_2}{g}$$

$$\beta = H_1 + 2\ \Delta H_1 + \frac{C_1V_1}{g}$$

$$\gamma = 1 + \frac{a'C_1}{g\Delta tA_f}$$

$$\Gamma = \frac{a'}{\Delta tA_f}\left(2\ \Delta H_1 + \frac{C_1V_1}{g}\right) + V_p$$

The positive root of equation (13) gives the flow rate through the pump after wave action.

The resulting velocity in the feed line is given by

$$V_{11} = \frac{1}{\gamma}\left(\frac{Q}{A_f} + \Gamma\right) \tag{14}$$

and the velocity in the discharge line is given by

$$V_{22} = \frac{Q}{A_d} \tag{15}$$

Pressure wave magnitudes and line pressures are computed from equations (2), (3), (4), and (5).

Injector. - The discharge line terminates with an injector, which forces propellant into the thrust chamber.

The relation between flow through the injector and the net head across the injector is closely approximated by a square law relation (ref. 4). This relation after wave action is

$$\frac{Q_2^2}{A_d^2} = B_I^2 (H_{11} - H_{22}) \tag{16}$$

The chamber pressure is a function of flow through the injector. They are related by a combustion dead time τ and a gas residence time θ_g. The relation between chamber pressure head H_c and injector flow rate Q_i (ref. 1) can be written in the form

$$\frac{dH_c}{dt} + \frac{H_c}{\theta_g} = \frac{c^*}{A_T g \theta_g} Q_i(t - \tau) \tag{17}$$

where c^* is the characteristic velocity, A_T is the throat area of the engine nozzle, and $Q_i(t - \tau)$ is the injector flow rate at a time $t - \tau$ in the past. In first order finite difference and wave-plan notation, equation (17) becomes

$$\frac{H_{22} - H_2}{\Delta t} + xH_{22} = yQ_b \tag{18}$$

where $x = 1/\theta_g$, $y = c^*/A_T g\theta_g$, Δt is the time increment between successive changes in the wave-plan analysis, and Q_b is the burning rate of the propellant ($Q_b = Q_1(t - \tau)$). A higher order backward finite difference approximation may be employed with little additional complication.

The injector is located in a dome that has compliance. The net compliance of the dome region is somewhat larger than the discharge line compliance. This locally high compliance is also taken into account analytically through the use of a hypothetical accumulator. The mathematical expression used is identical with the equation for the pump inlet accumulator (eq. (10)). In a real system using hydrocarbon fuel much of the compliance in this region is related to structural springiness; therefore, a linear accumulator will be used in this area (a' = constant). For hydrogen-fueled engines with cavitation occurring on the dome side of the injector because of heating, a nonlinear spring constant would be preferable.

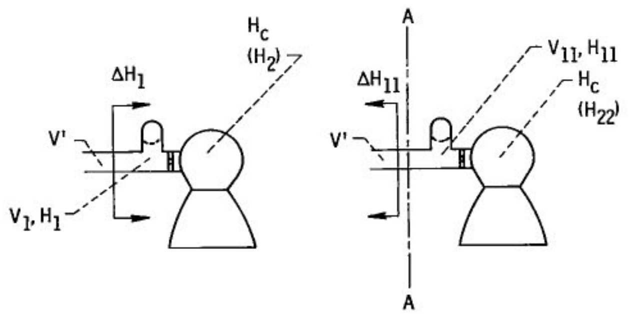


Figure 7. - Conditions at injector before and after wave action.

Figure 7 shows conditions before and after wave action at the injector. The continuity equation expressing the condition that the flow past boundary AA equals the flow through the injector plus the storage

for any small time Δt is

$$Q_2 = V_{11}A_d - \frac{\Delta \mathcal{V}}{\Delta t} \tag{19}$$

Substituting the relation for the accumulator pressure-volume curve (eq. (10)) and denoting the value of the slope as a'' give

$$Q_2 = V_{11}A_d - (H_{11} - H_1)\frac{a''}{\Delta t} \tag{20}$$

The line pressure after wave action is

$$H_{11} = H_1 + \Delta H_1 + \Delta H_{11} = H_1 + 2\,\Delta H_1 + \frac{C}{g}(V_1 - V_{11}) \tag{21}$$

Combining equations (16), (18), (20), and (21) and solving for the line velocity after wave action give the following equation, which is quadratic in V_{11}:

$$\alpha^2 V_{11}^2 + \left(\frac{B_I^2 C}{g} - 2\alpha\beta\right) V_{11} + \beta^2 - B_I^2(\gamma - H_{22}) = 0 \tag{22}$$

where

$$H_{22} = \frac{yQ_1 + (H_2/\Delta t)}{x + (1/\Delta t)}$$

$$\alpha = 1 + \frac{a''C}{g\,\Delta t A_d}$$

$$\gamma = H_1 + 2\,\Delta H_1 + \frac{CV_1}{g}$$

$$\beta = \frac{a''}{A_d\,\Delta t}\left(2\,\Delta H_1 + \frac{CV_1}{g}\right)$$

The positive root of equation (22) gives the discharge line velocity after wave action. The resulting injector flow rate is computed from equation (20).

Pump inlet perturbation suppressor. - A pump inlet perturbation suppression device is indicated in the model in figure 1 (p. 3) because of current interest in fix devices for reducing longitudinal structural oscillations of heavily loaded space launch vehicles. The function of such a device is to minimize the effect of flow perturbations induced in the feed system by structural motion. Two of the more common suppression devices being considered are suction line volume compensating devices (ref. 6) and high compliance devices.

The high compliance device can be represented by equation (10) by defining an appropriate function for the parameter a' to fit the particular type of hardware (or pressure-volume relation) chosen. The volume flow into the device must be included as an additional term in the continuity equation at the pump inlet (eq. (11)). For the special case where the losses and the inertia of the liquid in the standpipe are negligible and the compliance of the fix device is assumed to be of the same functional form as the cavitation induced pump inlet compliance, the net effect can be calculated with a single hypothetical accumulator by using the total compliance.

The volume-compensating device (e.g., a constant-volume bellows) can be represented analytically by a similar approach by adding to the continuity equation the following term for the volume flow Q_c into the compensator (ref. 6):

$$Q_c = kA_f V_p\left(1 - \frac{V_i}{V_p}\right) \tag{23}$$

where V_p is the pump motion velocity and V_i is the velocity of the other structural attachment point of the compensator. This point must be located in the forward part of the structure in order to get a large enough structural motion difference to operate the compensator. The kinematic factor k depends on the particular geometry of the compensator. Values of k for various types of compensators are given in reference 6. When the flow into the compensator is added to equation (11), the continuity equation becomes

$$V_{11} - V_p\left[1 + k\left(1 - \frac{V_i}{V_p}\right)\right] - \frac{a'}{\Delta t A_f}(H_{11} - H_1) = V_{22}\frac{A_d}{A_f} \tag{24}$$

Equation (24) is used instead of equation (11) for determining the form of equation (13) when calculating the pump flow rate Q when a volume compensating fix device is present.

Friction orifices. - Nonlinear (square law) orifices can be inserted throughout the system to simulate viscous effects. The analysis of this type of orifice subjected to pressure pulses is presented in reference 8.

Throttle valve. - A nonlinear (square law) throttle valve is located in the pump discharge line. The relation between pressure and flow is the same as that for a friction orifice and for an injector (eq. (16)). To throttle, the orifice coefficient B_T is varied with time in a prescribed manner. For simplicity, the orifice coefficient is varied linearly with time in the examples presented in this report.

Digital Programming

Time and position subscripts. - Combining the feed system component analyses into a single digital analysis of a propellant supply system is accomplished by introducing time and position subscripts. Through the use of these subscripts the time-independent equations developed for a point are extended to time-dependent system equations. (A detailed explanation of the use of the subscripts is included in ref. 8.) The subscripts are defined as follows:

L position subscript

J time subscript (time = $J \Delta t$)

In this notation the pressure head to the right of component L at time $t = J \Delta t$ is written as HR(L, J).

Each of the component analyses are written as a digital computer subroutine, which has the function of computing conditions at a system component at one instant of time in terms of conditions a small incremental time earlier, magnitudes of impinging waves, and occurrences that happend during the small time interval. The subscripted notation is generalized by defining variables as follows:

HR pressure head to the right of the component

HL pressure head to the left of the component

ΔHR wave to the right and going away from the component

ΔHL wave to the left and going away from the component

If the component introduces no change of area, the liquid velocity in the line does not change and is defined as V. If the line area changes at the component, the velocity will change across the component and is defined as

VI line velocity into component

VO line velocity out of component

The digital analysis consists of a finite difference type solution in which occurrences are computed at the end of small equal time intervals. The time interval is so chosen that wave travel times between adjacent components are always integer multiples of this time interval. The time subscript J refers to the integer number of time intervals since the initiation of the computations. The computations start with a known set of conditions at a discrete set of points and computes new conditions for these points for as long as desired.

The equations developed to analyze conditions at a component are directly applicable in the overall system analysis if the following substitutions are initially made:

$$H_1 = HL(L, J-1)$$

$$H_2 = HR(L, J-1)$$

$$\Delta H_1 = \Delta HR(L-1, J-KX)$$

$$\Delta H_2 = \Delta HL(L+1, J-KY)$$

$$V_1 = V(L, J-1)$$

or

$$V_1 = VI(L, J-1)$$

$$V_2 = VO(L, J-1)$$

where the quantities KX and KY refer to the number of time intervals necessary for a sonic disturbance to travel from the respective adjacent components.

The appropriate system component equations are then employed, and the corresponding outputs are as follows:

$$H_{11} = HL(L, J)$$

$$H_{22} = HR(L, J)$$

$$\Delta H_{11} = \Delta HL(L, J)$$

$$\Delta H_{22} = \Delta HR(L, J)$$

$$V_{11} = VI(L, J)$$

$$V_{22} = VO(L, J)$$

or

$$V_{11} = V(L, J)$$

The preceding transformations result in a set of general relations that compute conditions at system component L at time J in terms of conditions that were computed in a previous cycle.

Computer programs. - Digital computer subroutines were written for the principal system components. Subroutines were written for the tank outlet, the pump inlet station, the injector station, and the friction orifices. The throttle valve is a special case of the friction orifice subroutine in which the orifice coefficient is varied with time in a prescribed manner. A subroutine was not written for the fix device because in the examples of this report (see appendix B for system parameters) a high compliance device having the same functional form analytically as the pump inlet cavitation compliance was used. Thus the additional compliance could be incorporated into the accumulator at the pump inlet station. The four major subroutines are given in appendix C along with examples of the FORTRAN programs. These subroutines are incorporated into the computer program for the monopropellant feed system. A copy of a typical FORTRAN IV program for the propellant feed system model is given in appendix D.

Storage requirements. - Because of the large number of individual computations that must be made in the wave-plan analysis, it might be assumed that serious storage problems would be encountered when computing solutions for complex propellant systems. This, however, is not the case. The digital computations associated with the wave-plan analysis may be carried out (starting with initial conditions) to any desired time. Because the equations for each discontinuity are functions only of occurrences at the adjacent discontinuities, only the information for the maximum number of time intervals required for a sonic disturbance to travel from the farthest of the adjacent discontinuities need be stored.

At the injector the additional requirement exists that information must be stored for at least the number of time intervals equivalent to the dead time. After these requirements have been satisfied, the system conditions may be automatically reinitialized, and all necessary information will be available to the computer. Reinitializing allows a complex system composed of many discontinuities to be solved with relatively small computer storage capacity.

RESULTS AND DISCUSSION

The response of the hypothetical propellant feed system of figure 1 (p. 3) was computed in order to illustrate the capabilities of the wave-plan method. Transient and steady-state solutions were determined for both periodic and ramp input disturbances. The numerical values of the feed system parameters used in the example are given in appendix B. The major parameters are summarized in table I.

TABLE I. - MAJOR FEED SYSTEM PARAMETERS

Line lengths, ft	
Feed line	40
Discharge line	6
Line areas, ft^2	
Feed line	0.2
Discharge line	0.05
Acoustic velocities, ft/sec	
Tank	1000
Feed line	2000
Discharge line	3000
Propellant density, lb/ft^3	53
Mean flow rate, ft^3/sec	4
Mean velocity in feed line, ft/sec	20
Engine parameters, msec	
Combustion dead time	3
Gas residence time	1
Pump coefficients	
A	1208
B	668
C	-109
Structural motion input velocity amplitude, ft/sec	
Pump	0.8
Tank	0.32

Low-Frequency Periodic Inputs

The response of the feed system was determined for low-frequency (1 to 16 cps) sinusoidal inputs in the form of longitudinal oscillatory structural motion at the pump and tank inlet. This response is representative of the type of low-frequency disturbances sometimes encountered with spacecraft launch vehicles during the boost phase of flight. For simplicity the amplitude of the longitudinal structural oscillations was assumed to be independent of the magnitude of the thrust perturbations during the calculation for each frequency. To simplify the example further, the amplitudes of the imposed structural velocity perturbations at the pump and tank were held constant over the frequency range studied. To establish a basis for comparison, the first series of calculations were made with no accumulators (zero additional compliance) at the pump inlet or at the injector dome. The first series of results presented therefore pertain to a system in which the only compliance present is that due to the elasticity of the propellant and conduits.

Frequency dependence. - The frequency response of the propellant system was determined. Calculations were made for a range of frequencies from 5 to 16 cycles per second. This range included the quarter-wave frequency for the 40-foot feed line, which for this case is 12.5 cycles per second. The results are given in table II. All pressure heads were computed in feet of propellant.

The system responses were generally periodic functions resembling sinusoids. Some slight nonlinearities were evident at lower frequencies. Figure 8 shows several typical periodic responses.[1] The magnitudes of the negative and positive perturbations are approximately equal.

[1]The flow and pressure head scales used in figures 8, 10, 12, 17, 18, and 20 are arbitrary with the zero corresponding to the local mean value or to the original steady-state value of the parameter.

TABLE II. - AMPLITUDES OF PRESSURE AND FLOW PERTURBATIONS IN FEED SYSTEM

[No accumulators in feed system (a' = 0, and a'' = 0).]

Frequency, cps	Pressure head at feed line stations,[a] ft			Pump inlet pressure head, ft	Pump outlet pressure head, ft	Injector flow rate, ft^3/sec	$\frac{Q_i}{V_{BB}A_f}$	Dome pressure head, ft	Chamber pressure head, ft
	2	3	4						
5	16.1	23.1	28.8	35.9	28.4	0.037	0.23	25.7	10.9
7	26.2	37.3	47.4	55.9	44.2	.057	.36	39.2	16.9
9	41.9	58.3	73.4	82.7	65.5	.084	.52	59.8	25.2
10	52.7	73.6	90.1	101.2	79.5	.103	.64	72.9	30.8
11	66.7	93.3	111.9	123.1	96.1	.126	.79	88.2	37.3
12	80.8	112.2	132.7	142.0	110.7	.145	.91	101.8	42.9
12.5	87.5	120.5	141.4	148.5	115.5	.151	.94	106.4	44.9
13	93.6	123.8	148.0	149.7	117.6	.152	.95	107.9	45.4
13.5	94.1	126.4	146.9	148.5	116.7	.150	.94	106.1	44.7
14	93.1	124.7	142.1	142.2	111.2	.144	.90	101.5	42.8
14.5	88.8	122.2	134.2	133.8	105.0	.135	.84	94.6	40.2
16	76.9	101.9	108.7	99.7	78.9	.102	.64	70.7	30.1

[a]Feed line stations 2, 3, and 4 are located 24, 16, and 8 ft above the pump inlet station.

Pump suction pressure. - The performance of the pump is directly related to the inlet (suction) pressure. Because of this relation, pressure fluctuations at the pump are of prime importance, and the pump must be designed to operate satisfactorily under the most extreme pressure perturbation anticipated. As can be seen in table II, the maximum pressure perturbation (149.7 ft, 55 psi) occurred slightly above 13 cycles per second, which is near, but somewhat greater than the quarter-wave frequency (12.5 cps) for the 40-foot feed line. The frequency dependence of pump suction pressure perturbation amplitude can also be obtained from table II.

Minimum feed line pressure. - Relatively low pressures exist in the pump suction line. Therefore the lowest pressure that can occur in the feed line must be determined if the occurrence of feed-line cavitation is to be predicted. The instantaneous pressure at any point in the feed line is given by the sum of the steady-state or mean value and the perturbation value at that instant. The location in the feed line where this sum is a minimum must be determined, and the magnitude of the minimum value must be calculated. For the frequency range studied (table II), the maximum pressure perturbation amplitude occurs at the pump inlet, and the size of the perturbation amplitude decreases with distance up the feed line.

At any instant the local steady-state (or mean) pressure occurring in a vertical pipe line in a uniform acceleration field increases linearly from the tank to the pump. The pressure head gradient (feet of head per linear foot) due to acceleration of the center of mass of the vehicle in a gravitational field is

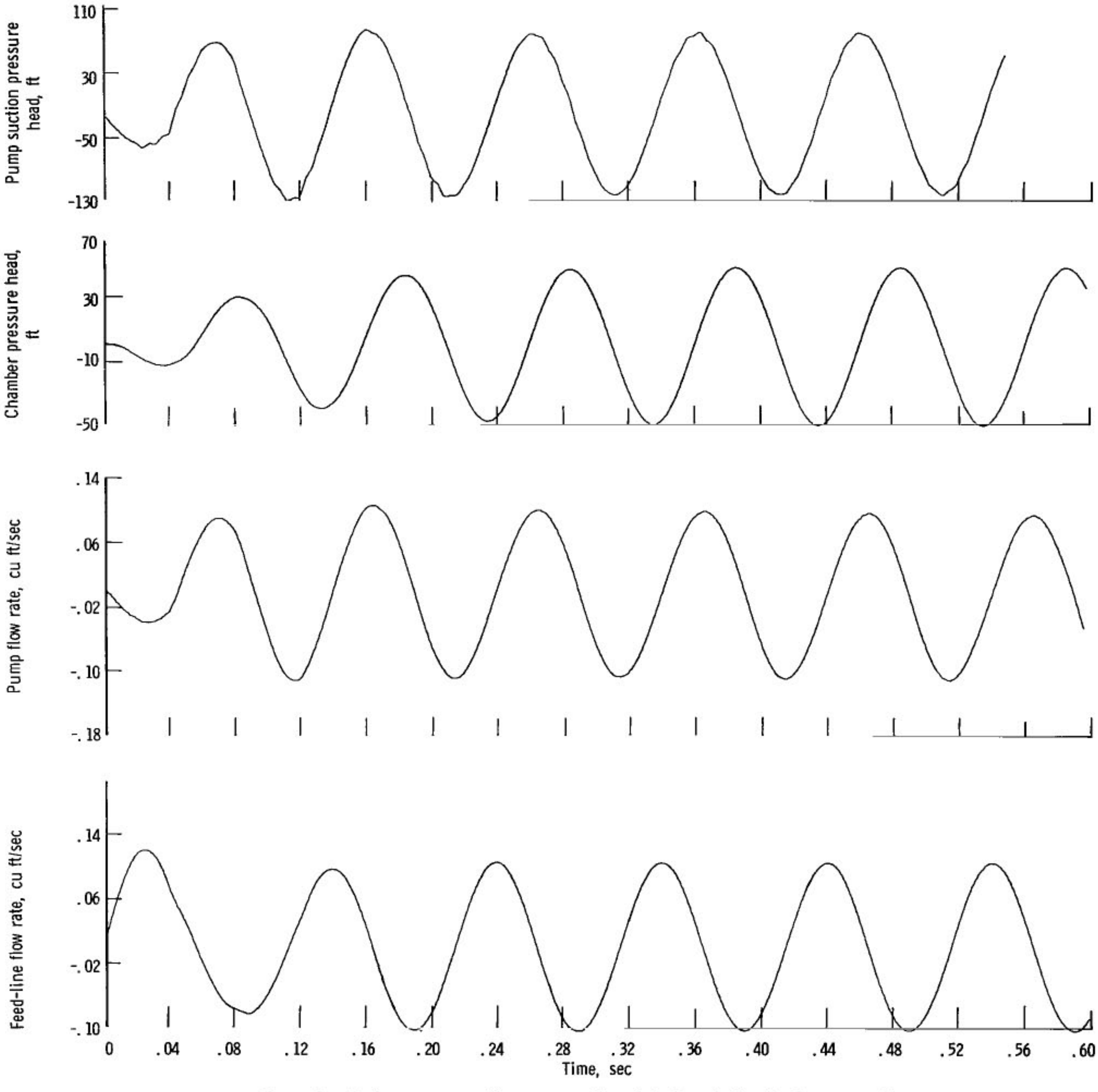


Figure 8. - System response with no accumulators (a' = 0 and a'' = 0). Frequency, 10 cps.

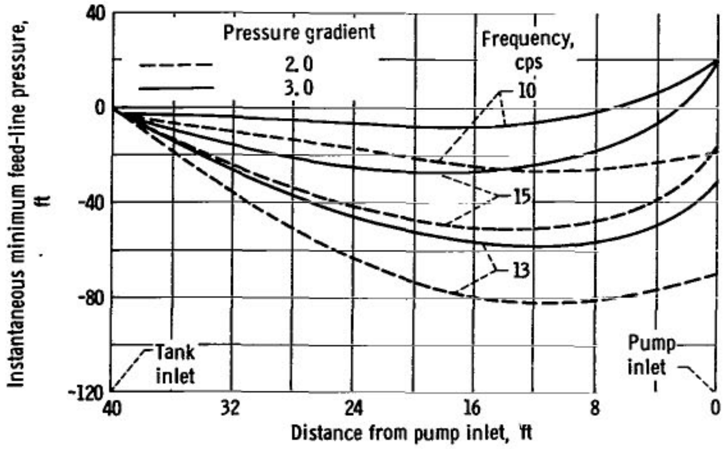


Figure 9. - Instantaneous minimum values of local pressure in feed line.

$$\frac{dH_g}{dz} = \frac{a_z + g}{g}$$

where a_z is the vertical acceleration of the center of mass of the vehicle. The pressure gradient due to friction losses is given by the Darcy equation as

$$\frac{dH_f}{dz} = -\frac{fV^2}{2gD}$$

Thus the overall steady (or mean) pressure gradient is

$$\frac{dH}{dz} = \frac{a_z + g}{g} - \frac{fV^2}{2gD}$$

For the system studied the pressure gradient due to friction is relatively small compared with the gravitational gradient and has a value of 0.4 foot of head per linear foot.

Figure 9 shows the variation in the local instantaneous minimum pressure along the feed line computed for two arbitrarily chosen overall steady gradients. The variation is given for three different structural oscillation input frequencies. The dashed line is for a slope of 2.0, which corresponds to an acceleration of the center of mass of 1.4 g's. The solid line is for a pressure slope of 3.0 corresponding to an acceleration of 2.4 g's. The lowest pressure generally occurs between 8 and 24 feet from the pump inlet and is considerably lower than the minimum value of pressure at the pump inlet.

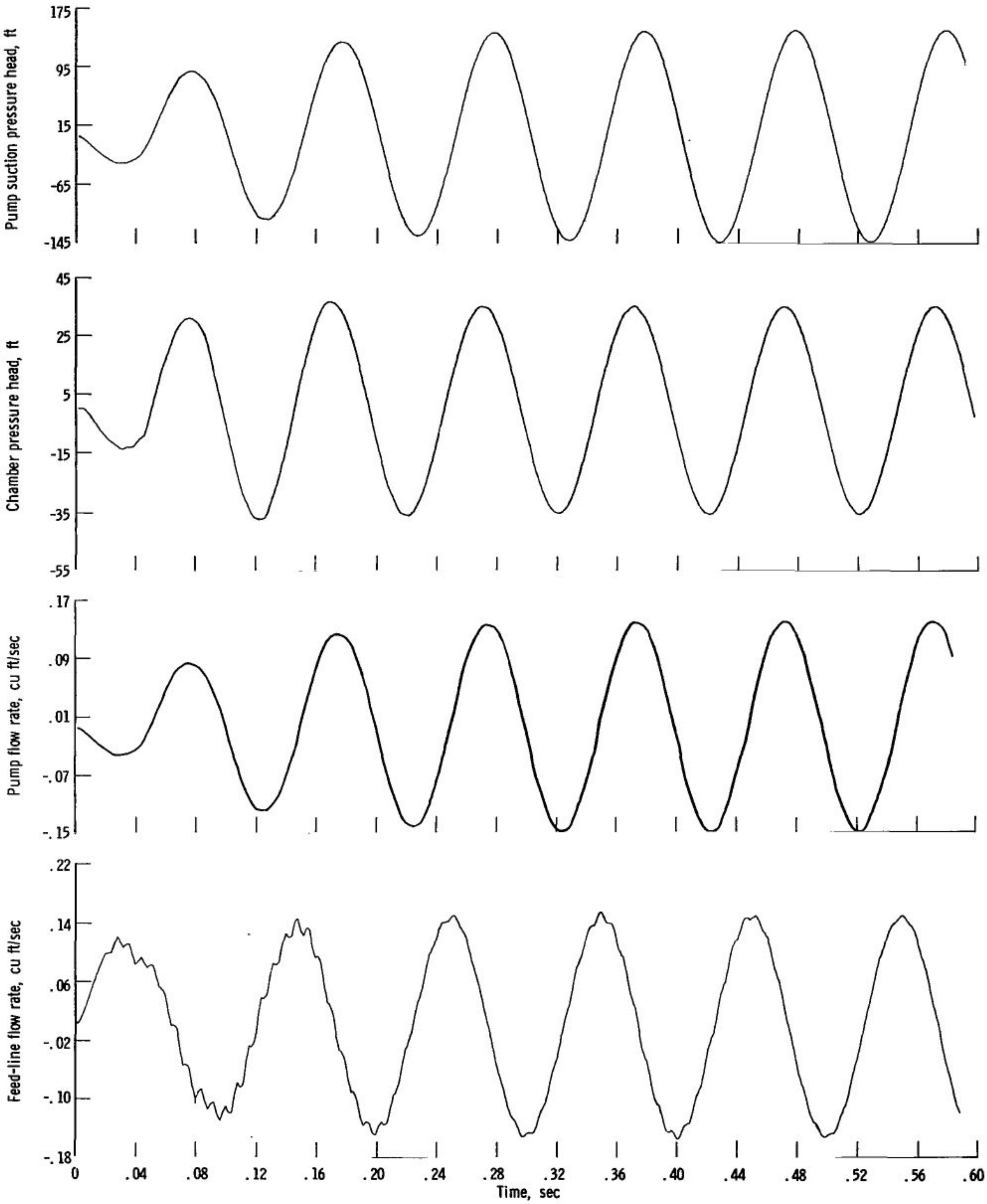


Figure 10. - System response with linear accumulator at pump inlet. Compliance of pump inlet accumulator, $a' = 2\times10^{-5}$; frequency, 10 cps.

This analysis points out that the location in the feed line of maximum negative pressure perturbation is not necessarily the point experiencing the lowest instantaneous net pressure.

Injector flow rate perturbation. - The injector flow rate perturbation is of particular importance because longitudinal structural oscillations of a launch vehicle are driven by thrust perturbations, which in turn are directly related to the propellant flow rate through the injector into the thrust chamber. As shown in table II, the frequency dependence of the amplitude of the propellant flow rate perturbation through the injector follows a pattern similar to the pump inlet pressure perturbation.

The amplitude of the ratio of the injector flow rate perturbation to the equivalent flow rate perturbation caused by the pump motion is also given in table II. This ratio reaches a maximum of 0.95 at the equivalent quarter-wave resonance condition (13 cps) of the feed system. At quarter-wave resonance the impedance to perturbation flow up the feed line toward the tank is very high. This high value indicates that as the pump moves up and down only a small part (5 percent) of the flow disturbance generated can travel up the line to the tank. Instead most (95 percent) of the disturbance travels through the pump and into the engine where it results in thrust perturbations.

Effect of Pump and Injector Compliance on Low-Frequency Oscillations

The accumulators (fig. 1, p. 3) at the pump inlet and injector were used to simulate locally high compliances at these stations. The effect of these compliances on pump inlet pressure perturbations, pump inlet flow perturbations (measured in the feed line just above the accumulator simulating inlet compliance), pump outlet flow perturbations, and thrust chamber pressure perturbations were calculated with the model.

Pump inlet with linear accumulator. - A quantitative indication of the effect of cavitation-induced compliance at the pump inlet was obtained with a constant compliance accumulator. The values used in the calculations were $a' = 0.2\times10^{-5}$, 1.0×10^{-5}, and 2.0×10^{-5} cubic feet per foot head.

Figure 10 shows the computed responses of various system parameters to sinusoidal longitudinal structural oscillations (compare with fig. 8). The calculations were carried out for a frequency of 10 cycles per second and an inlet compliance of 2×10^{-5} cubic feet per foot. The resulting perturbations in pump suction pressure, thrust chamber pressure, pump flow rate, and feed line flow rate are shown. The most noticeable effect of the accumulator is that system nonlinearities originating in the tank become apparent in the feed line flow rate. The nonlinear disturbance is transient in nature and eventually dies out. The frequency dependence of the amplitude of the pump inlet pressure perturbations is shown in figure 11 for the three compliance constants. The response curve with no accumulator in the line is also given for comparison. The general trend is for the resonant

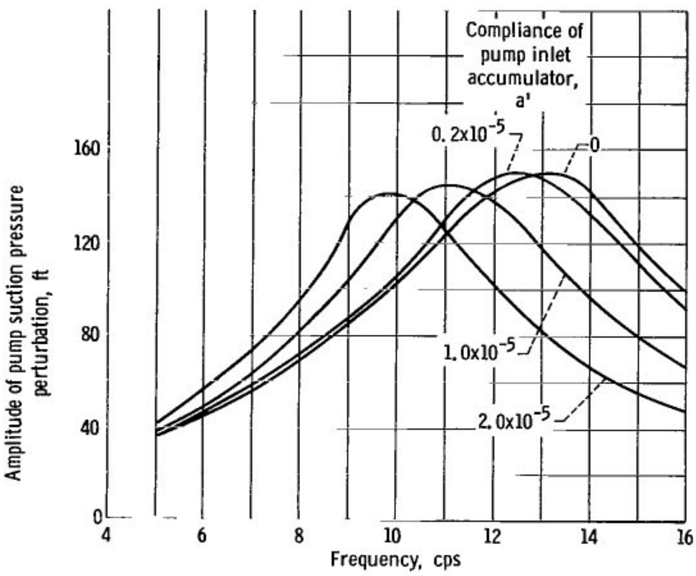


Figure 11. - Frequency response of pump suction pressure perturbation with linear accumulator at pump inlet.

H_o, ft	$\mathcal{V}_o$, ft^3
150	0.0005
150	.001
150	.004
200	.004
150	.01
150	.1

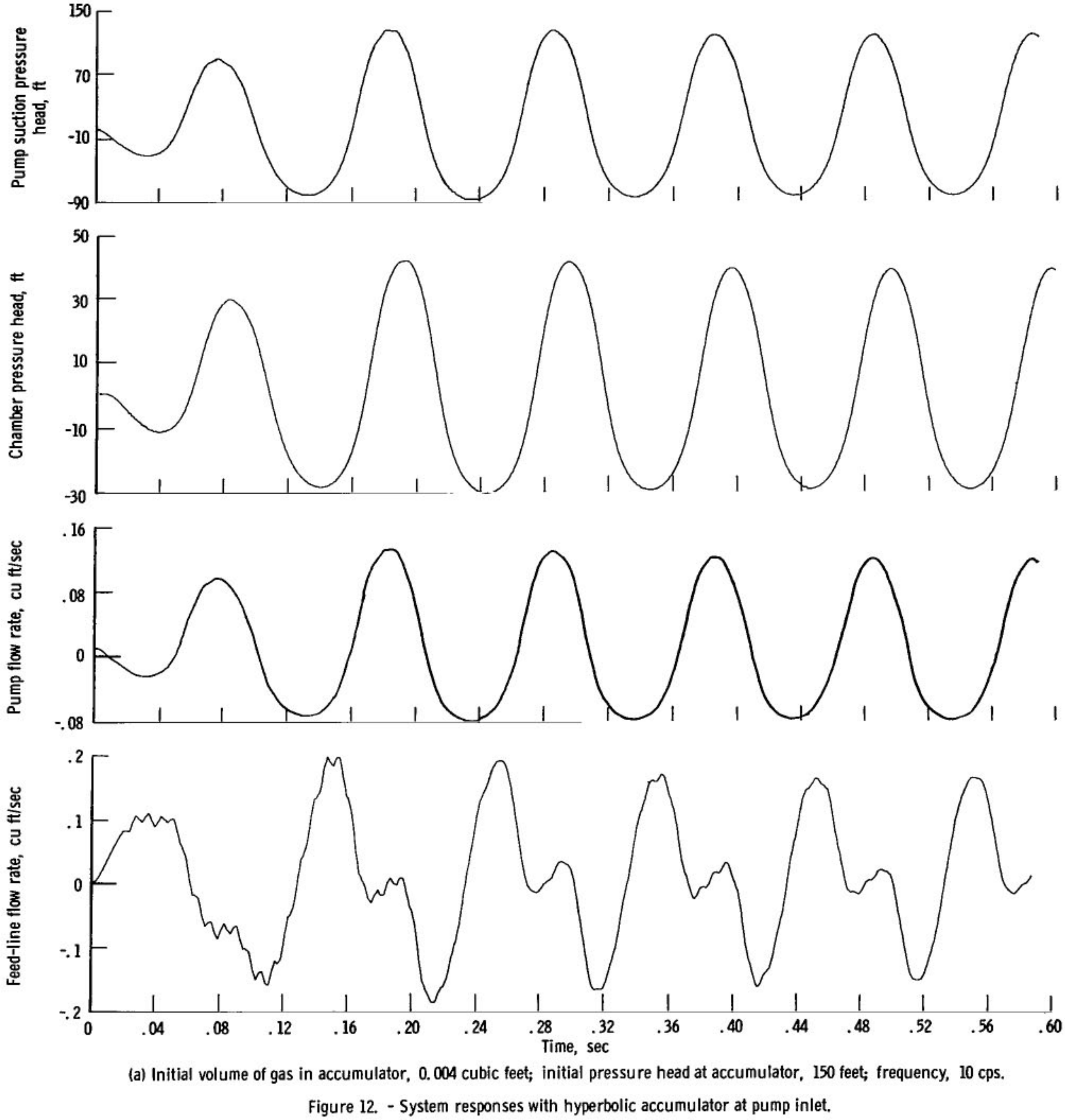
frequency to decrease as the inlet compliance is increased. Thus cavitation induced compliance in the pump inlet region tends to lower the resonant frequency of the suction line. Other parameters such as injector flow rate perturbation and thrust chamber pressure perturbation are affected in a similar manner.

Pump inlet with hyperbolic accumulator. - An indication of the effect of nonlinear compliance at the pump inlet was obtained by calculating the response of the system with the hyperbolic accumulator. This type of accumulator may be used to approximate pump inlet cavitation, a "high compliance type" fix device, or a combination of both.

Accumulators having the characteristics shown in the table above were analyzed.

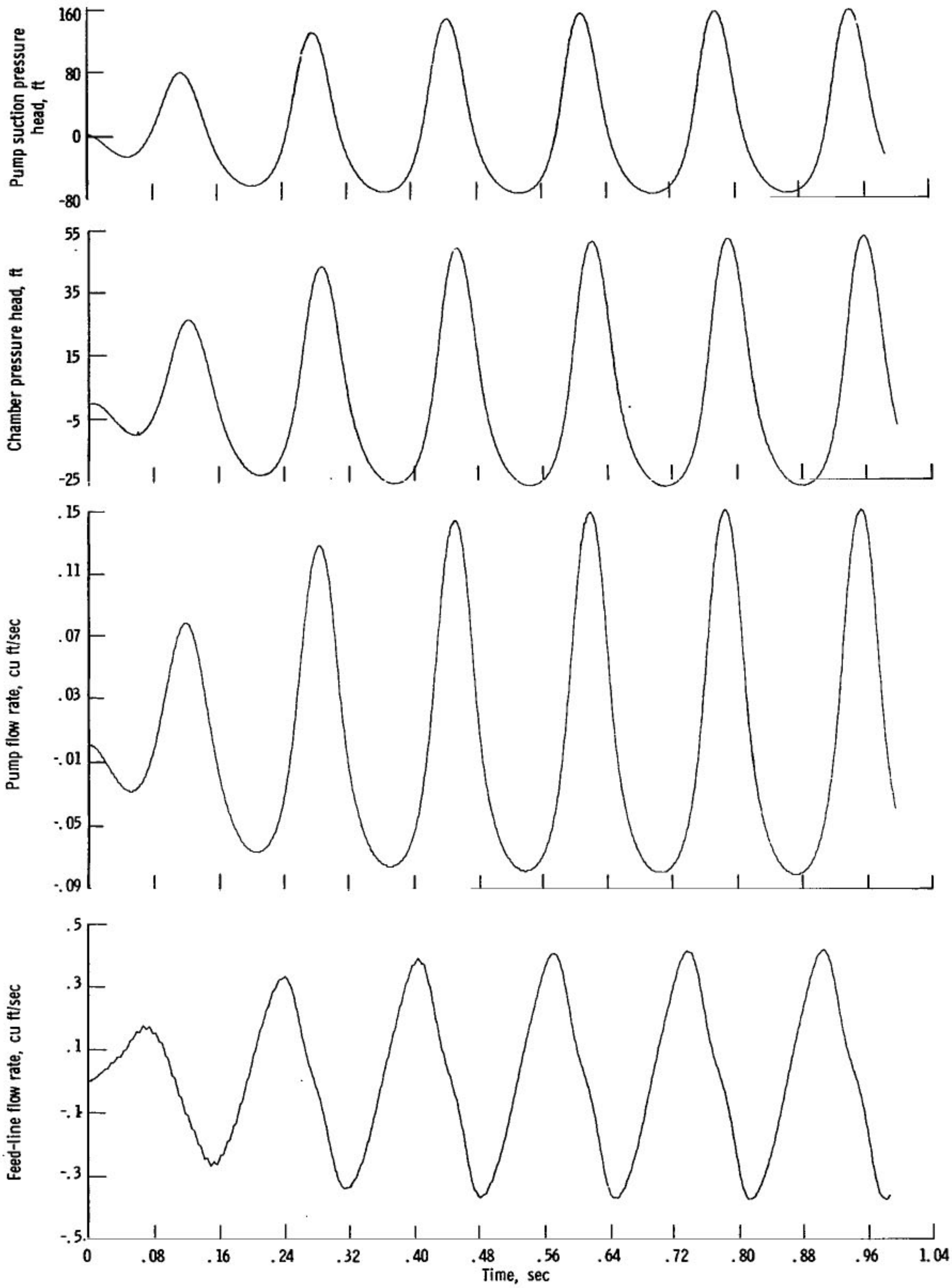
The smaller initial volumes can be considered to be representative of the cavitation-induced inlet compliance at pump suction pressures greater than or equal to H_{S1} (fig. 4, p. 7), whereas the larger initial volumes (0.01 and 0.1 ft^3) are representative of gas accumulator fix devices or alternately a combination of both compliances. The larger initial volumes are also typical of cavitation compliance for pumps operating at suction pressures lower than H_{S1} (fig. 4).

The nonlinear response of the system with hyperbolic accumulators at the pump inlet is indicated by the pressure-time and flow-rate-time traces shown in figure 12. The responses for system parameters at two arbitrary sets of conditions are shown. For the majority of cases calculated the responses of the pump suction head, chamber head, and pump flow rate are characterized by sharp high peaks and flat valleys. Occasionally the traces were quite irregular because of the presence of higher harmonics at a particular condition (near resonance). It was also noted that very large accumulator compliances tended to lengthen the time for the starting transient to decay. The feed line flow rate



(a) Initial volume of gas in accumulator, 0.004 cubic feet; initial pressure head at accumulator, 150 feet; frequency, 10 cps.

Figure 12. - System responses with hyperbolic accumulator at pump inlet.



(b) Initial volume of gas in accumulator, 0.01 cubic feet; initial pressure head at accumulator, 150 feet; frequency, 6 cps.

Figure 12. - Concluded.

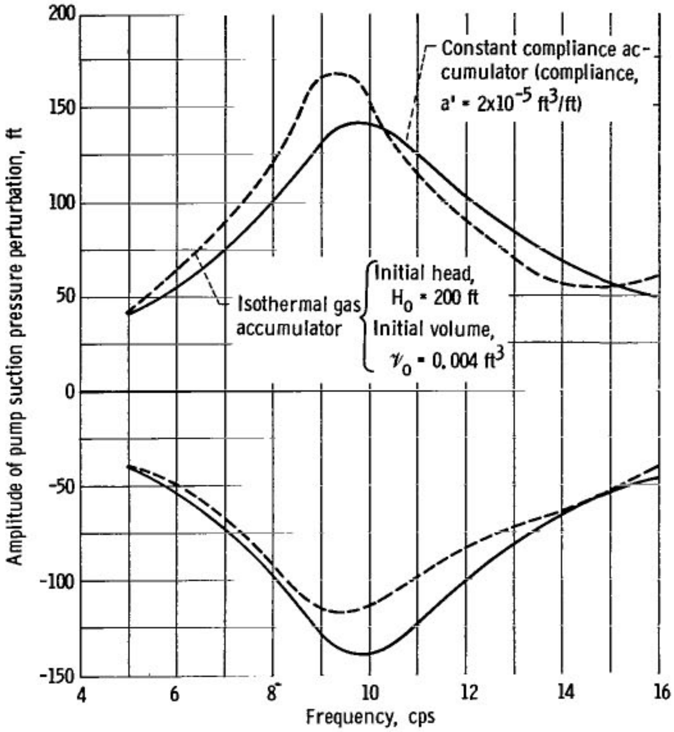
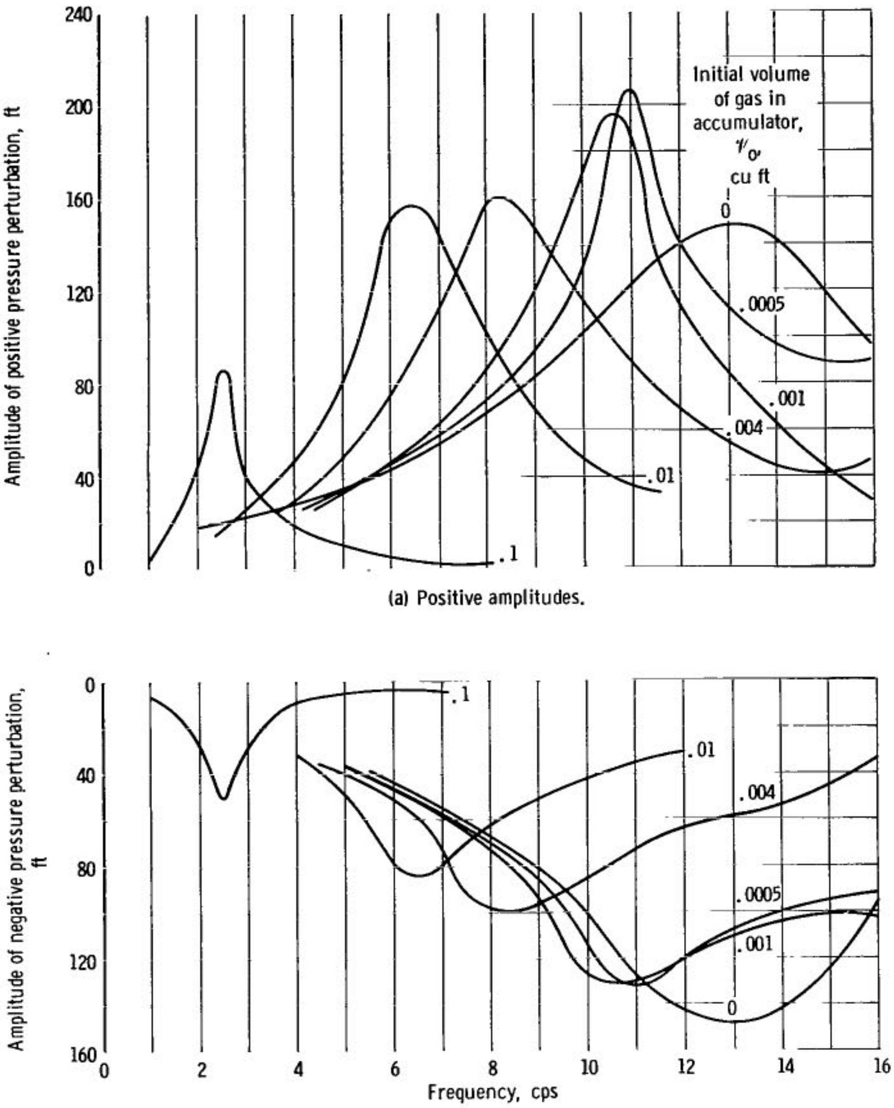


Figure 13. - Nonlinear response of pump suction pressure with gas accumulator.

above the accumulator displayed no predominant pattern but was characterized by nonlinear behavior.

The frequency response of pump inlet pressure perturbation amplitude for a pump having a hyperbolic accumulator at the inlet is compared in figure 13 with the response of a pump with a linear (or constant compliance) accumulator at the inlet. The hyperbolic accumulator has the same compliance (2×10^{-5} ft^3/ft) at mean line conditions as the linear accumulator. The hyperbolic accumulator is represented by an isothermal gas accumulator with a volume of 0.004 cubic feet (6.9 in.^{3}) at a mean line pressure of 200 feet. The curves for the positive and the negative amplitudes as a function of frequency are mirror images for the linear accumulator. The curves for the hyperbolic accumulator, however, are not mirror images because, as shown in figure 12, the positive amplitude is greater than the negative amplitude. This results from the hyperbolic form of the compliance, which increases with decreasing pressure and decreases with increasing pressure.

The effect of pump inlet compliance on the suction line resonant frequency was determined by examining the computed system response with hyperbolic accumulators of various sizes at the pump inlet. In figure 14, the positive and negative amplitudes of the perturbation pressures are plotted as functions of frequency for accumulators of various sizes. The accumulators have the various indicated volumes at an initial pressure of 150 feet. Figure 14 shows that increased pump inlet compliance has the effect of lowering the suction line resonant frequencies, as was noted for the linear accumulator results (fig. 11, p. 23). Because of the hyperbolic functional form of the accumulators, the positive amplitudes at resonance are greater than the resonant value for the system with no accumulator. Only at very large compliances does the positive amplitude at resonance become equal to or smaller than the resonant value for the system with no accumulator. The negative amplitude at resonance is always smaller than the no-compliance case and decreases steadily with increasing values of inlet compliance. The peak to peak amplitude at resonance first increases with the addition of compliance (V_o = 0.0005) then



(a) Positive amplitudes.

(b) Negative amplitudes.

Figure 14. - Pump suction pressure perturbation with hyperbolic accumulator.

steadily decreases with increasing compliance. Figure 14 demonstrates that any increase in pump cavitation occurring during the boost phase of flight would lower the resonant frequency of the feed system. Further, a pure compliance (no inertia, or resistance) fix device placed at the pump inlet would have the function of lowering the pump suction line resonant frequency.

Injector dome compliance. - The propellant feed system response was determined for three values of compliance of the dome. The compliances are assumed to remain constant, and the values used include 5×10^{-6}, 2.5×10^{-6}, and 0.5×10^{-6} cubic feet per foot.

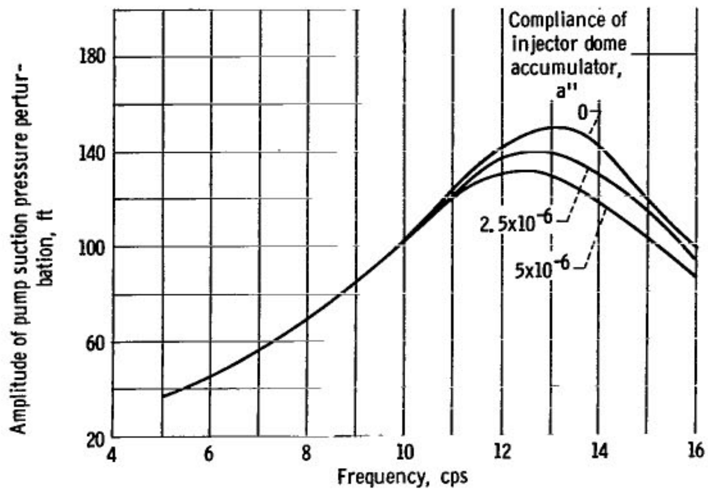


Figure 15. - Frequency dependence of pump suction pressure perturbation with accumulator at injector dome.

The frequency dependence of the pump inlet pressure perturbation is shown in figure 15. The curve for a compliance of 0.5×10^{-6} is not shown because it follows the zero compliance curve so closely. This value is close to the value for the line compliance. The accumulator at this point in the system has no great effect at longitudinal structural oscillation frequencies. The critical frequency and maximum amplitudes are both slightly lowered. The accumulator has a relatively small effect at this location because the linearized value of the injector impedance (at the mean flow condition) is only about one-half of the characteristic impedance of the discharge line.

Transient Response of System

Throttling. - The digital model was employed to determine the propellant system response to throttling. Throttling transients were introduced at the pump discharge line throttle valve (fig. 1, p. 3). With the system running at a steady-state operating condition the throttling transient was introduced by initiating a linear variation of the orifice coefficient from B_{T1} to B_{T2} in a short period. This variation is illustrated in figure 16.

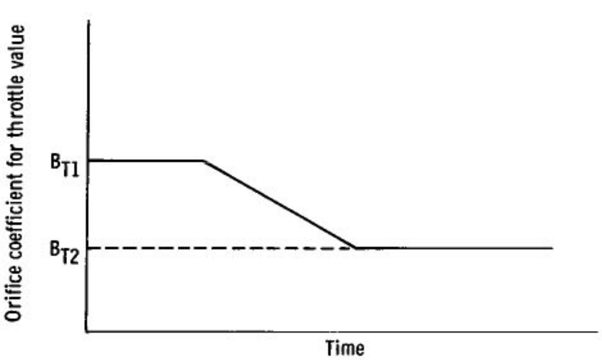


Figure 16. - Variation of orifice coefficient with time during throttle operation.

When the system was subjected to transient inputs due to throttling it responded in one of two ways. The system flow and pressure usually underwent short duration

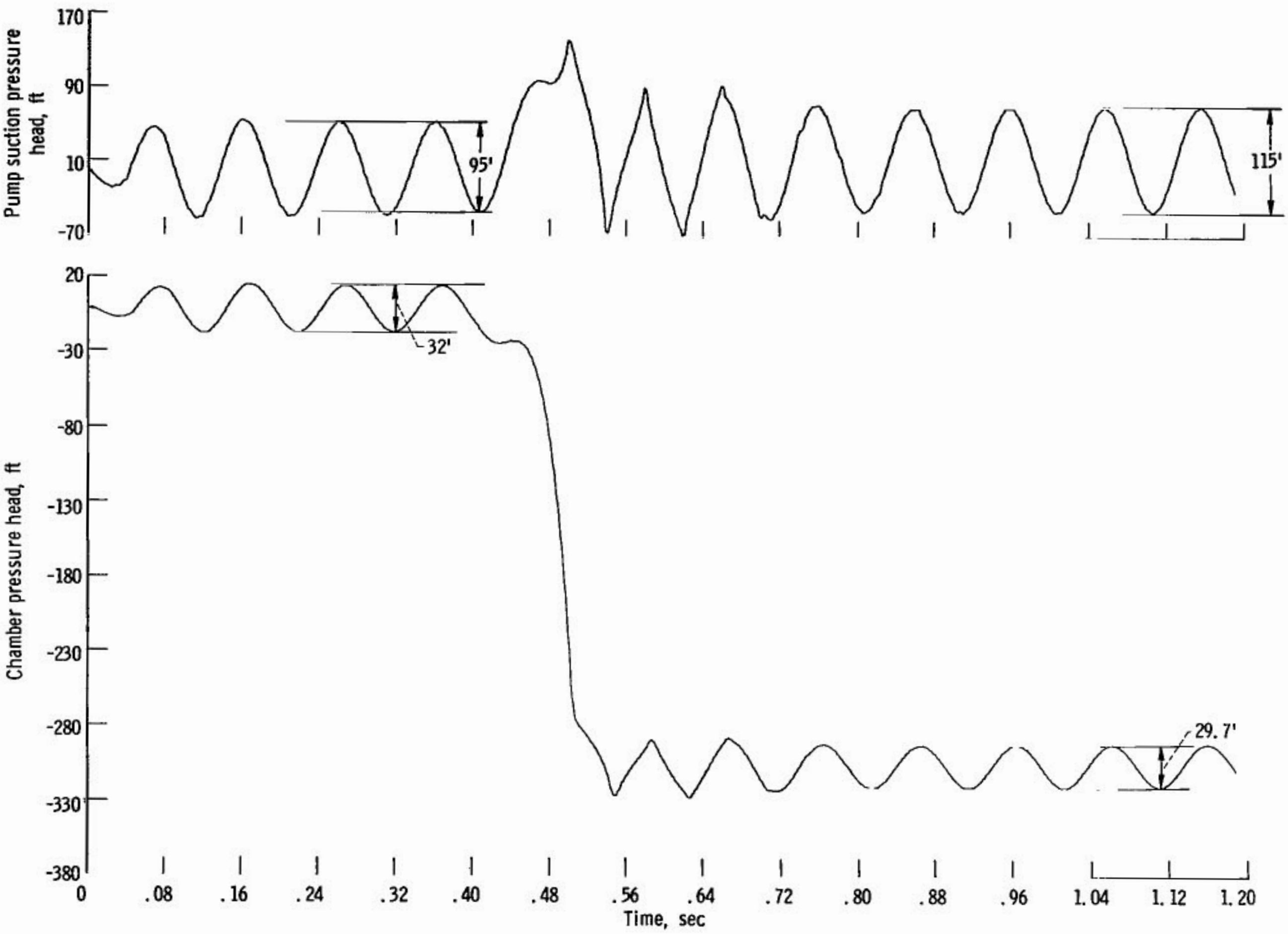
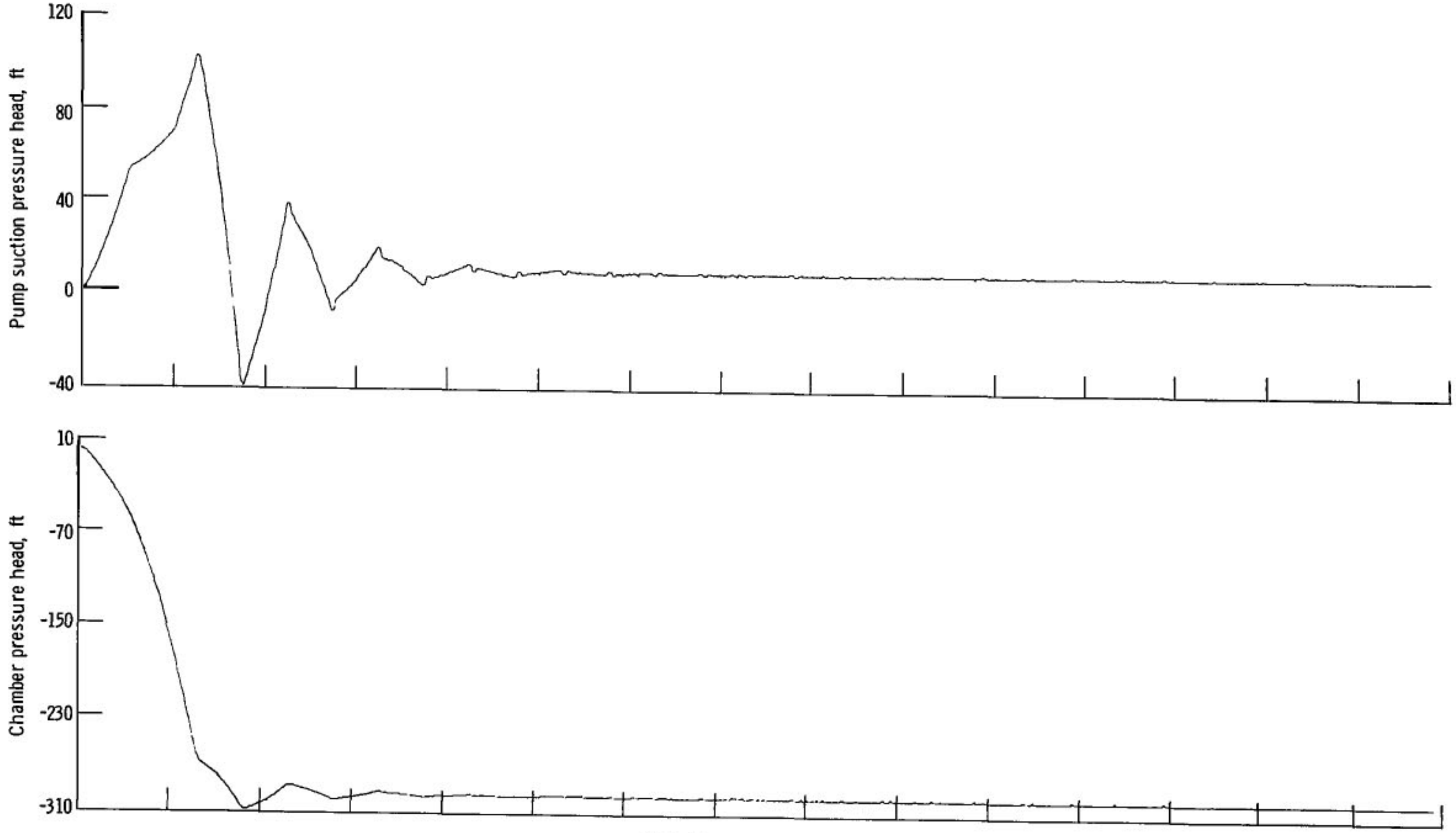
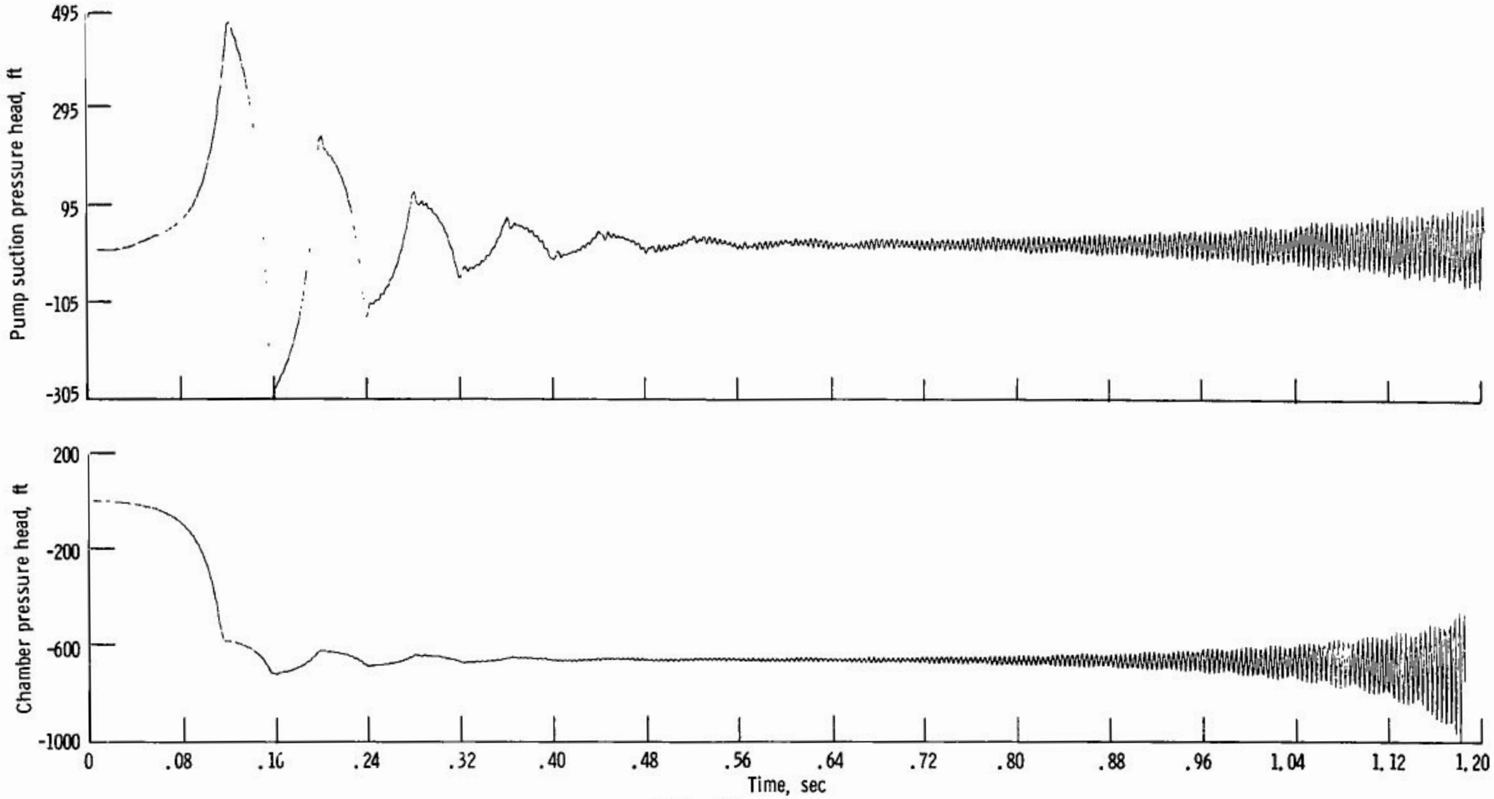


Figure 17. - System response to throttling with longitudinal motion.



(a) Stable response. $B_{T2} = 2.0$.



(b) Unstable response. $B_{T2} = 1.0$.

Figure 18. - System response to throttling with no longitudinal motion.

disturbances that eventually died out. The system then returned to the original steady-state values or to a new steady-state condition when a permanent change in the system operating conditions had been introduced. Alternately there were conditions under which the initial throttling transient disturbance did not fully damp out before the system went into an unstable mode of operation (chugging) because it ended up in an unstable operating region.

Figures 17 and 18 contain examples of throttling transients. In figure 17 the system was throttled while oscillating longitudinally. (No lateral motions were assumed to be present.) Both the pump suction pressure and the thrust chamber pressure perturbations were approximately steady-state sinusoids before the throttling was initiated. Following the short transient period, the pump suction and thrust chamber pressures quickly returned to a stable steady-state sinusoidal oscillation at a new mean level and with a new perturbation amplitude. The amplitude of the structural motion input was held constant during the computational period. The change in perturbation amplitudes was caused by a shift in system operating conditions.

Figure 18 shows responses for a system that was throttled from an initially steady operating condition (no structural oscillations). In figure 18(a) the throttling introduced a transient that quickly died out, and the system ended at a new stable operating point. Figure 18(b) shows a condition where the system was throttled to a greater degree. In this case the system mean operating parameters changed significantly, and the system went into an unstable high frequency chugging oscillation before the throttling transient had died out. The calculations were terminated at the end of the pressure-time traces shown because the system was unstable.

Gimbal snubbing. - During flight one or more engines of a rocket may use gimbaling action to keep the missile on the desired flight path. When the thrust chamber and feed system pumps are mounted so that they both move during the gimbaling action, transients are introduced into the system. During the initial gimbaling maneuver, the accelerations are low because of engine inertia, and no noticeable transients would be introduced into the feed system; however, under some circumstances the engine gimbal motion may be stopped suddenly by snubbing action. A sudden stop would introduce a transient into the feed system. In this example the pump-engine assembly will initially be assumed to be moving relative to the tank outlet station at the maximum velocity (2 ft/sec) obtained during the gimbaling motion. The motion of the engine-pump assembly is then assumed to be linearly snubbed from 2 feet per second to rest in 40 milliseconds (fig. 19). The effect of the snubbing action on the pump suction pressure head is shown in figure 20. The snubbing action was initiated at 10 milliseconds, and the system response was

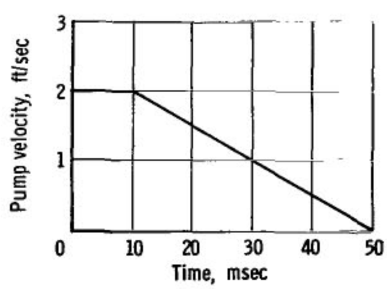


Figure 19. - Pump velocity during gimbaling-snubbing maneuver.

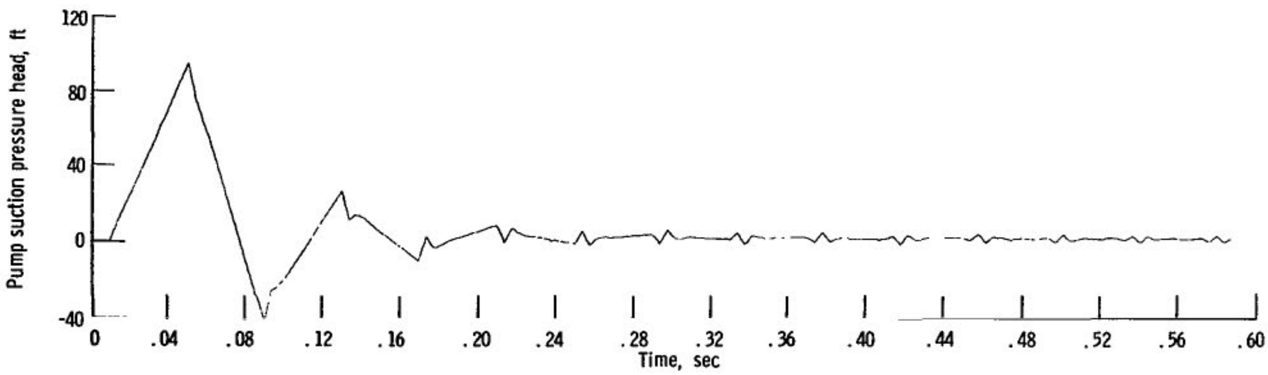


Figure 20. - System response to rapid snubbing of pump motion resulting from engine gimbaling.

calculated out to 600 milliseconds. Figure 20 shows that in this severe case snubbing caused an increase in pressure of 97 feet above and 41 feet below the mean condition. The pressure transient quickly dies out, however.

CONCLUDING REMARKS

The simple propellant feed system analyzed in this report does not correspond to any operational propellant supply system. It is, however, fundamentally similar in many respects to operational systems and serves to illustrate how the wave-plan method can be applied to analyze unsteady flow in a rocket propellant feed system.

The fact that a monopropellant (or one side of a bipropellant) feed system was analyzed is not meant to imply that the method is limited to such a system. The method can readily be extended to a bipropellant system by coupling two feed systems together at the thrust chamber. This coupling is done through the relation between thrust chamber pressure and the fuel and oxidizer weight flow rates; however, for some analytic purposes the cross-coupling effects on the individual feed system responses are small, and the simpler monopropellant model can be used to explore the effect of varying basic system parameters and geometries.

In the calculated examples for the feed system response to structurally imposed perturbations, the amplitude of the structural motion at any input point was assumed to remain constant during the calculation; that is, no feedback was provided between combustion chamber pressure (and, therefore, thrust) perturbations and the structural oscillations. Similarly no feedback was provided between local feed system pressures (at the pump inlet for example) and the structure. However, the model can be readily combined with structural dynamic equations to provide the necessary feedback. Thus, in cases

where the structural dynamics equations are available the wave-plan method of analyzing the feed system can be incorporated into a nonlinear stability analysis.

Because of the present lack of dynamic data on turbopumps the steady-state operating curve was used to determine the dynamic resistance of the pump. In addition, the analysis was limited to the design operating region of the pump. Again, these assumptions were not basic to the method. A very complex pressure-flow map or tabulated dynamic data for the pump can be easily read into the digital program when available. Further, no coupling between the pump and turbine was put into the examples. Including the turbine would require a method of characteristics solution on the gas side as well. This extension, although entirely feasible, was considered beyond the scope of this report.

In conclusion it should be emphasized that the calculated response of a fluid system is very specific and highly dependent on the particular hydraulic and structural dynamic characteristics of the system analyzed. It is, therefore, very difficult and usually dangerous to make conclusions that apply to systems in general. The best approach is to analyze the specific hardware at the operating conditions of interest.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, March 23, 1966.

APPENDIX A

SYMBOLS

A, B, C — coefficients for static net head-discharge curve for pump

a, b, c — coefficients for quadratic functions

A_d — cross-sectional area of discharge line, ft^2

A_f — cross-sectional area of feed line, ft^2

A_T — throat area of engine nozzle, ft^2

A_t — cross-sectional area of tank, ft^2

a' — compliance of pump inlet accumulator, ft^3/ft

a'' — compliance of injector dome accumulator, ft^3/ft

B_F — frictional orifice coefficient, $(ft/sec^2)^{1/2}$

B_I — orifice coefficient for injector, $(ft/sec^2)^{1/2}$

B_T — orifice coefficient for throttle valve, $(ft/sec^2)^{1/2}$

C_d — wave velocity in discharge line, ft/sec

C_f — wave velocity in feed line, ft/sec

C_t — wave velocity in tank, ft/sec

C_1 — wave velocity in line preceding component, ft/sec

C_2 — wave velocity in line following component, ft/sec

C* — characteristic velocity, ft/sec

D — line diameter, ft

f — friction factor

g — acceleration due to gravity, ft/sec^2

H — pressure head, ft

ΔH — pressure head change across wave or component, ft

H_c — chamber pressure head, ft

H_f — pressure head due to friction losses, ft

H_g — pressure head due to acceleration of gravity, ft

H_o — initial pressure head at accumulator, ft

H_1 — pressure head to left of component before wave action, ft

ΔH_1 — wave impinging from left of component before wave action, ft

H_2 — pressure head to right of component before wave action, ft

ΔH_2 — wave impinging from right of component before wave action, ft

H_{11} pressure head to left of component after wave action, ft

ΔH_{11} wave leaving component on left side after wave action, ft

H_{22} pressure head to right of component after wave action, ft

ΔH_{22} wave leaving component on right side after wave action, ft

H_S, H_{S1} pump suction pressure head, ft (see fig. 4)

ΔH_p pressure head rise across pump, ft

k compensator kinematic factor

L_d length of discharge line, ft

L_f length of feed line, ft

Q volume flow rate, ft^3/sec

Q_b burning rate of propellant, $Q_b = Q_1(t - \tau)$, ft^3/sec

Q_c volume flow into compensator, ft^3/sec

Q_1, Q_i injector discharge at time equal to dead time in past, ft^3/sec

Q_2 injector discharge after wave action

R_1 ratio of feed line to tank area

t time, sec

V velocity, ft/sec

V_{BB} amplitude of pump sinusoidal structural motion velocity, ft/sec

V_i linear velocity of structure at compensator attachment point, ft/sec

V_p linear velocity of pump due to structural motion, ft/sec

V_t linear velocity of tank due to structural motion, ft/sec

V_1 velocity in line to left of component before wave action, ft/sec

V_2 velocity in line to right of component before wave action, ft/sec

V_{11} velocity in line to left of comnent after wave action, ft/sec

V_{22} velocity in line to right of component after wave action, ft/sec

$\mathscr{V}$ volume, ft^3

$\mathscr{V}_o$ initial volume of gas in accumulator, ft^3

x, y injector delay and dead time coefficients

θ_g gas residence time, sec

τ dead time, sec

APPENDIX B

SYSTEM PARAMETERS USED IN EXAMPLE

The hypothetical propellant feed system is shown in figure 1 and consists of a tank and a pump connected by a feed line containing a noncompensating bellows. The pump discharges into a discharge line and into a nonlinear (square law) injector.

In order to make a digital analysis of the response of the system it is necessary to assign numerical values to all parameters. The effect of certain key parameters on the response of the system can be evaluated by varying their numerical values.

The propellant tank and line parameters were assigned the following values:

Wave velocity in tank, C_t, ft/sec	1000
Wave velocity in feed line, C_f, ft/sec	2000
Wave velocity in discharge line, C_d, ft/sec	3000
Length feed line, L_f, ft	40
Length discharge line, L_d, ft	6
Area tank, A_t, ft^2	20
Area feed line, A_f, ft^2	0.2
Area discharge line, A_d, ft^2	0.05

The propellant was assumed to have a density of 53 pounds per cubic foot. The mean propellant flow rate was 4 cubic feet per second at the design operating condition. The injector pressure drop was approximately 40 percent of the dome pressure. An injector coefficient B_I equal to 2.8 was used.

Thrust chamber coupling with the feed system was provided by using a chamber pressure that is a function of the injector flow rate and related in time by a time delay factor (gas residence time) and a dead time. The combustion dead time was chosen to be 3 milliseconds. The gas residence time θ_g was taken as 1 millisecond, and the quantity $c^*/A_t g \theta_g$ was approximately 370 000 per square foot.

The following pump coefficients were employed in equation (9): $A = 1208$, $B = 668$, and $C = -109$.

Viscous losses in the tank were very small and were neglected. The total friction loss in the feed line at mean line conditions ($Q = 4\ ft^3/sec$) was assumed to be 16 feet. The corresponding loss in the discharge line was assumed to be 100 feet. The higher value is due to the high velocities present in the discharge line. These losses were distributed over four friction orifices located in the feed line and one located in the discharge line.

At the pump and tank, coupling to the structure was provided by forced motion of

these components along the vehicle vertical axis. This structural motion was assumed to be sinusoidal, and the amplitude of the tank motion was 40 percent of the pump motion amplitude. The amplitude of the structural velocity perturbation at the pump was taken to be 0.8 feet per second in the subsequent examples. This value is 4 percent of the mean flow velocity in the feed line. For simplicity, the same structural motion amplitudes were used at all frequencies.

To complete the definition of mean operating conditions for the system the height of fluid in the tank and the pressure at some point in the system must be designated. Because of the long propellant line length, the dynamic response of this system has only a small dependence on propellant height in the tank. Since critical frequencies usually occur near burnout, a constant level of fluid in the tank of 2 feet was assumed and was used for the subsequent calculations. The propellant level could have been treated as a time-dependent variable, but the calculations were carried out for only a short interval, and the added complexity was not warranted. The dynamic response of the system as formulated is independent of mean pressures except for cases when local compliances are pressure dependent. For these cases the mean pressure at the local station is defined in the RESULTS AND DISCUSSIONS section of the report.

APPENDIX C

COMPUTER SUBROUTINES

With the use of the subscripting notation four digital computer subroutines were written. These subroutines perform the following operations:

(1) Equate the nonsubscripted initial conditions to their subscripted counterparts

(2) Solve the appropriate equations for a new set of conditions

(3) Equate the newly computed conditions to their subscripted counterparts

The four principal subroutines are as follows:

(1) Subroutine tank

(2) Subroutine pump

(3) Subroutine injector

(4) Subroutine friction orifice

Each subroutine requires the definition of the time counter J and the position counter L before it is entered. In addition, the following quantities must be defined for each subroutine:

Subroutine Tank:

VT	average velocity of tank over time interval
KX	number of time increments to nearest component in tank
KY	number of time increments to nearest component in feed line
C1	wave velocity in tank
C2	wave velocity in feed line
R	ratio of feed line to tank area

Subroutine Pump:

VP	average velocity of pump over time increment
KX	number of time intervals to nearest component in feed line
KY	number of time intervals to nearest component in discharge line
C1	wave velocity in feed line
C2	wave velocity in discharge line
A, B, C	pump characteristic coefficients
AP	compliance of pump inlet spring-piston accumulator
R	ratio of feed line to discharge line area

Subroutine Injector:

BI	injector coefficient
KY	number of time intervals to nearest component in discharge line

C wave velocity in discharge line

APP compliance of injector spring-piston accumulator

Subroutine Friction Orifice:

BF friction orifice coefficient

C wave velocity in line

K number of time intervals to adjacent component

The FORTRAN IV program for each subroutine including tables relating subscripted to nonsubscripted variables is as follows:

```
      SUBROUTINE TANK (V1,V2,H1,H2,DH1,DH2,V11,V22,H11,H22,DH11,DH22,VT)
      COMMON/BOX1/AP,APP,AD,AF,DT,X,Y,AT ,BO
      COMMON/BOX3/A,B,C,G,BF,ERROR
      COMMON/BOX2/C1,C2,R
C         V1        VI(L,J-1)
C         V2        VO(L,J-1)
C         H1        HL(L,J-1)
C         H2        HR(L,J-1)
C        DH1        DHR(L-1,J-KX)
C        DH2        DHL(L+1,J-KY)
C        V11        VI(L,J)
C        V22        VO(L,J)
C        H11        HL(L,J)
C        H22        HR(L,J)
C        DH11       DHL(L,J)
C     DH22      DHR(L,J)
C        VT         VT(J)
      AA=(1.-R*R)/(2.*G)
      BB=C1*R/G+C2/G-VT*(R-R*R)/G
      CC=H2+2.*DH2-H1-2.*DH1-C2*V2/G-C1*V1/G+C1*VT*(1.-R)/G-VT*VT*
     1(1.-R)*(1.-R)/(2.*G)
      V22 = V2
      DO 5II=1,20
    2 ER = (AA*V22*V22+BB*V22+CC)/(2.*AA*V22+BB)
      V22 = V22-ER
      IF(ABS(ER)-ERROR) 1,1,5
    5 CONTINUE
      ERROR=1000.
      WRITE(6,100) V22,ER
  100 FORMAT(2E15.8)
    1 V11 = VT*(1.-R)+V22*R
      DH11 = DH1+C1/G*(V1-V11)
      DH22 = DH2+C2/G*(V22-V2)
      H11 = H1+DH1 +DH11
      H22 = H2+DH2+DH22
      RETURN
      END
```

```
      SUBROUTINE INJECT(H1,H2,V1,Q1 ,DH1,H11,H22,V11,Q2 ,DH11)
      COMMON/BOX1/AP,APP,AD,AF,DT,X,Y,AT ,BI
      COMMON /BOX3/ A,B,C,G,BF,ERROR
      DOUBLE PRECISION APPD,CD,GD,DTD,DH1D,V1D,H1D,V11D,BID,H22D,
     1ALPHA,BETA,GAM,AA,BB,CC,ERD
C     H2          HR(L,J-1)
C     V1          V (L,J-1)
C     H1          HL(L,J-1)
C       Q1=    Q(L,J-K5)
C       DH1=   DHR(L-1,J-K)
C     V11         V (L,J)
C     H11         HL(L,J)
C     H22         HR(L,J)
C     DH11        DHL(L,J)
C       Q2=   Q(L,J)
      H22=(Y*Q1+H2/DT)/(X+1./DT)
      APPD=APP
      CD=C
      GD=G
      DTD=DT
      DH1D=DH1
      V1D=V1
```

```
      H1D=H1
      BID=BI
      H22D=H22
      ALPHA=1.D0+APPD*CD/(GD*DTD)
      BETA=APPD/DTD*(2.D0*DH1D+CD*V1D/GD)
      GAM=H1D+2.D0*DH1D+CD/GD*V1D
      V11D=V1D
      AA = ALPHA*ALPHA
      BB=BID*BID*CD/GD-2.D0*ALPHA*BETA
      CC=-BID*BID*(GAM-H22D)+BETA *BETA
      NN=0
    1 ERD=(AA*V11D*V11D+BB*V11D+CC)/(2.D0*AA*V11D+BB)
      V11D=V11D-ERD
      ER=ERD
      V11=V11D
      IF(ABS(ER)-ERROR) 2,2,6
    6 NN=NN+1
      IF(NN-50) 1,1,7
    7 WRITE(6,101)V11,ER
      ERROR=1000.
  101 FORMAT(2E16.8)
    2 DH11 = DH1+C/G*(V1-V11)
      H11 = H1+DH1+DH11
      Q2=V11*AD-APP*AD/DT*(H11-H1)
      RETURN
      END


      SUBROUTINE FOR(H1,DH1,DH2,H2,VI,H11,DH22,DH11,H22,V11)
C     SUBROUTINE FRICTION ORFICE
C      H1       HL(L,J-1)
C      DH1      DHR(L-1,J-K)
C      DH2      DHL(L+1,J-K)
C      H2       HR(L,J-1)
C      V1       V(L,J-1)
C      H11      HL(L,J)
C      DH11     DHL(L,J)
C      DH22     DHR(L,J)
C      H22      HR(L,J)
C      V11      V(L,J)
      COMMON/BOX3/ A, B, C,G,BF,ERROR
      BB = 2.*BF*BF*C/G
      CC=-(H1+2.*DH1+2.*C/G*V1-(H2+2.*DH2))*BF*BF
      IF(CC) 3,3,2
    2 BB = -BB
      CC = -CC
    3 V11 = V1
      DO 6II=1,20
    4 ER = (V11*V11+BB*V11+CC)/(BB+2.*V11)
      V11 = V11-ER
      IF (ABS(ER)-ERROR ) 5,5,6
    6 CONTINUE
      ERROR=1000.
      WRITE(6,100) V22,ER
  100 FORMAT(2E15.8)
    5 DH11 = DH1+C/G*(V1-V11)
      H11 = H1+DH1+DH11
      DH22 = DH2+C/G*(V11-V1)
      H22 = H2+DH2+DH22
      RETURN
      END
```

```
      SUBROUTINE PUMP(VP,V1,V2,H1,H2,DH1,DH2,V11,V22,H11,H22,DH11,DH22 )
      DOUBLE PRECISION  QD,AA,BB,CC,C2D,BD,GD,C1D,AFD,APD,DTD,H2D,
     1DH2D,DH1D,V1D,V2D,AAD,VPD,ADD,ERD
      COMMON/BOX1/AP,APP,AD,AF,DT,X,Y,AT ,BO
      COMMON/BOX2/C1,C2,R
      COMMON/BOX3/A,B,C,G,BF,ERROR
C     V1   =  VI(L,J-1)
C     V2   =  VO(1,J-1)
C     H1   =  HL(L,J-1)
C     H2   =  HR(L,J-1)
C     DH1   =  DHR(L-1,J-KX)
C     DH2   =  DHL(L+1,J-KY)
C     V11   =  VI(L,J)
C     V22   =  VO(1,J)
C     H11   =  HL(L,J)
C     H22   =  HR(L,J)
C     DH11   =  DHL(L,J)
C     DH22   =  DHR(L,J)
      Q = V2*AD
      H1D=H1
      C2D = C2
      BD = B
      GD = G
      C1D = C1
      AFD = AF
      APD = AP
      DTD = DT
      H2D = H2
      DH2D = DH2
      DH1D = DH1
      V1D = V1
      V2D = V2
      AAD = A
      VPD = VP
      QD = Q
      ADD = AD
      H22D = H22
      AA=C
      BB = BD-C2D/(GD*ADD)-C1D/(GD*AFD*(1.D0+APD*C1D/(GD*DTD)))
      CC = -(H2D+2.D0*DH2D-H1D-2.D0*DH1D-(C1D*V1D+C2D*V2D)
     1/GD-AAD+C1D/GD*(APD/DTD*(2.D0*DH1D+C1D*V1D/GD)+VPD)/
     2(1.D0+APD*C1D/(GD*DTD)))
      DO 5II=1,20
    2 ERD = (AA*QD*QD+BB*QD+CC)/(2.D0*AA*QD+BB)
      QD = QD-ERD
      ER = ERD
      Q = QD
      IF(ABS(ER)-ERROR) 3,5,5
    5 CONTINUE
      ERROR=1000.
      WRITE(6,100) V22,ER
  100 FORMAT(2E15.8)
    3 V22 = Q/AD
      V11 = ((APD*(2.D0*DH1D+C1D*V1D/GD))/DTD+VPD+QD/AFD)/
     1(1.D0+APD*C1D/(GD*DTD))
      DH11=DH1+C1/G*(V1-V11)
      DH22=DH2+C2/G*(V22-V2)
      H11=H1+DH1+DH11
      H22=H2+DH2+DH22
      RETURN
      END
```

APPENDIX D

EXAMPLE OF PROGRAM FOR MONOPROPELLANT FEED SYSTEM

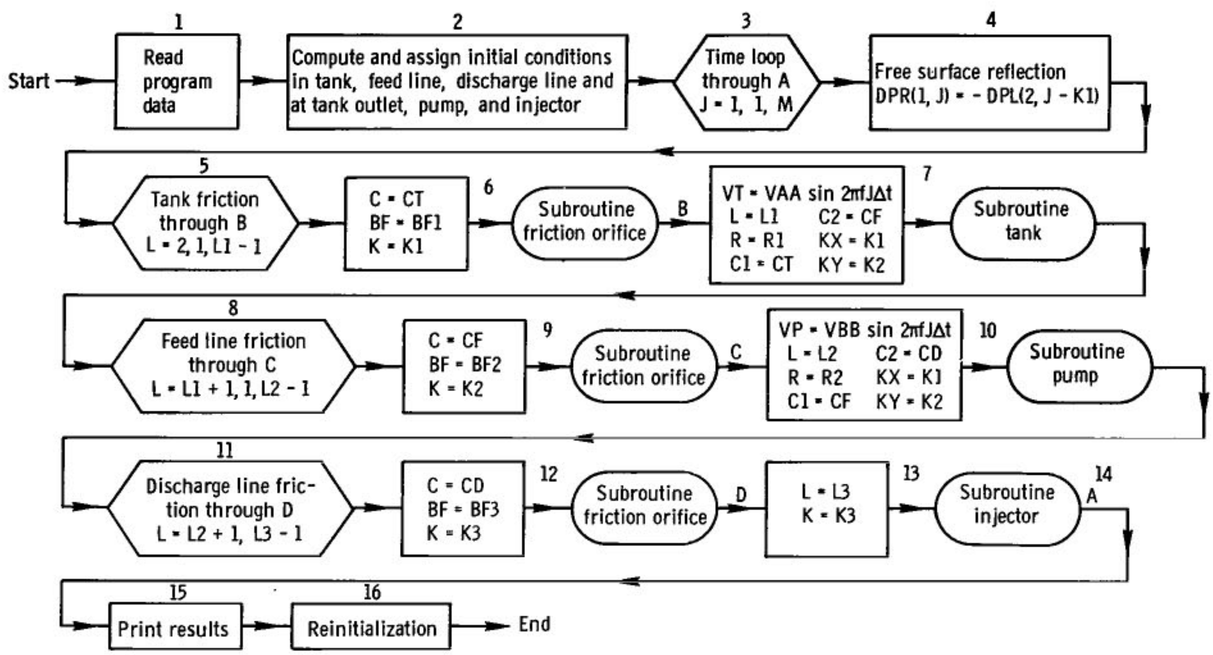


Figure 21. - Flow chart for monopropellant system.

Computer Program

The formulation of the computer program follows the technique outlined in reference 8. A flow chart of the program for linear compliance at the pump and injector is given in figure 21. Very minor changes are necessary if the flow chart is used for hyperbolic accumulations. An explanation of the steps shown in the flow chart in figure 21 is as follows; each of the following steps is correspondingly numbered in the flow chart:

(1) Read in the following initial conditions and physical constants as data:

L1 number of tank outlet component

L2 number of pump component

L3 number of injector component

K1 number of time intervals for wave travel between friction orifices in tank

K2 number of time intervals for wave travel between friction orifices in feed line

K3	number of time intervals for wave travel between friction orifices in discharge line
K4	largest of K terms
K5	number of time intervals for combustion dead time
CT	sonic velocity in tank, ft/sec
CF	sonic velocity in feed line, ft/sec
CD	sonic velocity in discharge line, ft/sec
VAA	amplitude of sinusoidal velocity perturbation of tank inlet, ft/sec
VBB	amplitude of sinusoidal velocity perturbation of pump, ft/sec
R1	ratio of feed line to tank area
R2	ratio of feed line to discharge line area
AF	area feed line, ft^2
A, B, C	static pump curve coefficients
BI	coefficient of injector, $ft^{1/2}/sec$
ΔT	working time interval, sec
M	number of time intervals computations are to be carried out
S	frequency, cps
BF1	friction orifice coefficient in tank, $ft^{1/2}/sec$
BF2	friction orifice coefficient in feed line, $ft^{1/2}/sec$
BF3	friction orifice coefficient in discharge line, $ft^{1/2}/sec$
Q	mean-line discharge, ft^3/sec
PR1	ullage pressure, ft
DH1	pressure change between tank friction orifices, ft
DH2	pressure change between feed line friction orifices, ft
DH3	pressure change between discharge line friction orifices, ft
AP	pump inlet compliance/feed line area
APP	injector dome compliance/discharge line area
x	reciprocal of gas residence time, 1/sec
y	chamber constant, $1/ft^2$

(2) Compute initial conditions and make assignments:

Initial Conditions		Assignments
V1(L2, 0)	L = 2, 1, L3	L = 1, 1, L3
V0(L2, 0)	HL(L, 0)	J = -K4, 1, 0
VI(L3, 0)	HR(L, 0)	DHR(L, J) = 0
V0(L3, 0)	V(L, 0) $(L \neq L2, L3)$	DHL(L, J) = 0

(3) Set up time loop and carry out computations for M cycles.

(4) Assign a negative reflection for all waves reflected from liquid surface.

(5) Set up loop for tank friction.

(6) Set conditions and enter friction subroutine for tank.

(7) Set conditions and enter subroutine tank.

(8) Set up loop for feed line friction.

(9) Set conditions and enter friction subroutine for feed line.

(10) Set conditions and enter subroutine pump.

(11) Set up loop for discharge line friction.

(12) Set conditions and enter friction subroutine for discharge line.

(13) Set conditions and enter subroutine injector.

(14) End time loop.

(15) Print values of pressures and flows throughout the system.

(16) Reinitialize as desired after a minimum of K4 intervals.

The following numerical values for the computer program data were used in the calculations:

L1 = 2	CT = 1000 ft/sec
L2 = 7	CF = 2000 ft/sec
L3 = 9	CD = 3000 ft/sec
K1 = 1	R1 = 0.01
K2 = 4	R2 = 4
K3 = 1	AF = 0.2 ft^2
K4 = 4	VAA = 0.32 ft/sec
K5 = 3	VBB = 0.80 ft/sec
A = 1208	BI = 2.8
B = 668	T = 0.001 sec

$C = -109$	BFI^2 = negligible
$Q = 4\ ft^3/sec$	$BF2 = 10$
$PR1 = 172.29$ ft	$BF3 = 8$
$DH1 = 0$	
$DH2 = 0$	
$DH3 = 0$	

AP = 0, 0.0001, 0.00005, 0.00001

APP = 0, 0.0001, 0.00005, 0.00001

S = 5, 7, 9, 10, 11, 12, 13, 14, 16

A listing of the FORTRAN IV program for linear compliance at the pump and injector is as follows:

[2] The tank friction was assumed to be negligible, therefore, no friction orifices were inserted in the tank.

```
      COMMON/BOX1/AP,APP,AD,AF, T,X,Y,AT ,BO
      COMMON/BOX2/C1,C2,R
      COMMON/BOX3/A,B,C,G,BF,ERROR
      G=32.
      DIMENSION PL(20,101),PR(20,101),          V(20,101),VI(2,101),
     1VO(2,101),DPR(20,133),DPL(20,133),VA(101),VT(101),QQ(20,133) ,
     2YY2(6000)

C          READ PROGRAM CONSTANTS

    1 READ (5,101) L1,L2,L3,K1,K2,K3,K4,K5,M,L5,L6,L7,L8,KODE2
      LL4=L2+1
      LL5=L3-1
      LLL=L1-1
      LL3=L2-1
      KKK = 100
      KK2 = KKK+1
      READ(5,100) T,S,BO,  CT,CF,CD,       VAA,VBB,R1,R2,AF,A,B,CC,BF1,BF
     12,BF3,DH1,DH2,DH3,   Q,AP,APP,ERROR,X,DP
  100 FORMAT(8F10.5)
  101 FORMAT(15I5)
  600 FORMAT(5E16.4)
      IF(KODE2) 2222,2221,2222
 2222 READ(5,100)S
      IF(S.EQ..0) GO TO 1
 2221 LL2=L1+1
      MN = M
      KL2=-1
      AD=AF/R2
      AT=AF/R1
      PR(1,1)=(Q/AD/BO)**2-225.+9. *(Q/(AD*BF3))**2

C          INITIAL CONDITIONS IN TANK 2 TO L1-1

      IF(LLL-2) 90,91,91
   91 DO 60 L=2,LLL
      V(L,1)=Q/AT
      PL(L,1)=PR(L-1,1)+DH1
   60 PR(L,1)=PL( L   ,1)-(V(L,1)/BF1)**2

C           INITIAL CONDITIONS AT TANK-L1

   90 PL(L1,1)=PR(LLL,1)+DH1
      VI(1,1)=Q/AT
      VO(1,1)=Q/AF
      PR(L1,1)=PL(L1,1)+(VI( 1,1)*VI( 1,1)-VO( 1,1)*VO( 1,1))/(2.*G)

C           INITIAL CONDITIONS IN FEEDLINE- L1+1 TO L2-1

      DO 61 L=LL2,LL3
      V(L,1)=Q/AF
      PL(L,1)=PR(L-1,1)+DH2
   61 PR(L,1)=PL(L,1)-( V(L,1)/BF2)**2

C           INITIAL CONDITIONS AT PUMP- L2

      L=L2
      PL(L,1)=PR(L-1,1)+DH2
      VI(2,1)=Q/AF
      VO(2,1)=Q/AD
      PR(L,1)=PL(L,1)+A+B*Q+CC*Q*Q
```

```
C          INITIAL CONDITIONS IN DISCHARGE LINE- L2+1 TO L3-1

      DO 62 L=LL4,LL5
      PL(L,1)=PR(L-1,1)+DH3
      V(L,1)=Q/AD
   62 PR(L,1)=PL(L   ,1)-(V(L,1)/BF3)**2

C          INITIAL CONDITIONS AT INJECTOR- L3

      L=L3
      PL(L,1)=PR(L-1,1)+DH3
      DO 63 J=1,K5
   63 QQ(L,J)=Q
      V(L3,1)=Q/AD
      PR(L,1)=PL(L,1)-(V(L,1)/BO)**2
      PEL=PR(L,1)
      Y=X*PEL/Q

C         WRITE PROGRAM CONSTANTS AND INITIAL CONDITIONS

      WRITE(6,115)
  115 FORMAT(1H1,6X,1HT,11X,1HS,11X,2HBO,10X,2HCT,10X,2HCF,10X,2HCD,10X
      WRITE(6,110) T,S,BO,CT,CF,CD,VAA,VBB
      WRITE(6,116)
  116 FORMAT(1HO,6X,2HAT,10X,2HAF,10X,2HAD,10X,1HA,11X,1HB,11X,1HC,11X,
     13HBF1,9X,3HBF2,9X,3HBF3,9X,3HDH1)
     13HBF1,9X,3HBF2,9X,3HBF3,9X,3HDH1)
      WRITE(6,117)
      WRITE(6,110) DH2,DH3,Q,AP,APP,ERROR
  117 FORMAT(1HO,6X,3HDH2,9X,3HDH3,9X,1HQ,11X,2HAP,10X,3HAPP,9X,5HERROR)
      WRITE(6,110)AT,AF,AD,A,B,CC,BF1,BF2,BF3,DH1
      WRITE(6,111)L1,L2,L3,K1,K2,K3,K4,K5,M,L5,L6 ,L7,L8
  111 FORMAT(4HOL1=,I5,5X,3HL2=,I5,5X,3HL3=,I5,5X,3HK1=,I5,5X,3HK2=,I5,
     15X,3HK3=,I5,5X,3HK4=,I5,5X,3HK5=,I5,5X,2HM=,I5/  4HOL5=,I5,5X,3HL6
     1=,I5,5X,3HL7=,I5,5X,3HL8=,I5)
      WRITE(6,502)
      WRITE(6,501)(PL(L,1),PR(L,1),V(L,1),L,L=1,L3)
  502 FORMAT(1HO,5X,2HPL,13X,2HPR,13X,1HV,10X,1HL)
  501 FORMAT(3F15.5,I5)
      WRITE(6,503)          VO(1,1),VI(1,1),VO(2,1),VI(2,1)
  503 FORMAT(4HOVO=,G12.4,5X,3HVI=,G12.4)
      WRITE(6,120)
  120 FORMAT(1HO,7X,5HPL-L1,4X,5HPR-L1,4X,5HPL-L2,4X,5HPR-L2,4X,5HPL-L3,
     1   4X,5HPR-L3,4X,5HVI-L2,4X,5HVI-L1,4X,4HV-L3,5X,5HPR-L5,4X,5HPR-L
     26, 4X,5HPR-L7,4X,2HBO,4X,3HVGP)


C         FORTRAN          ACTUAL MEANING
C        PL(L ,1)     =    PL(L ,0)
C        PR(L ,1)     =    PR(L ,0)
C        DPR(L,1)=DPR(L,-K4)
C        DPL(L,1)=DPL(L,-K4)
C        V(L,1)=V(L,0)
C        VI(1,1)     =   VI(L1,0)
C        VO(1,1)     =   VO(L1,0)
C        VI(2,1)     =   VI(L2,0)
C        VO(2,1)     =   VO(L2,0)
       DO 2  L=1,L3
       DO 2  J=1,K4
       DPL(L,J) = .0
```

```
    2 DPR (L,J) = .0
      CON = 2.*3.141593*S*T
      JK=K4-K1
      JL=K4-K2
      JM=K4-K3
      JN=K4-K5
      XX1=PR(L1,1)
      X2=PL(L1,1)
      X3=QQ(L3,1)/AD
      X4=PR(L5,1)
      X5=PL(L2,1)
      X6=PR(L2,1)
      X7=VI(1,1)
      X9=V(L3,1)
      X10=PL(L3,1)
      X11=PR(L2,1)
      X12=PR(L3,1)
      X13=PR(L5,1)
      X14=PR(L6,1)
      X15=PR(L7,   1)
      X16=PR(L8,   1)
      PR(L3,1)=PR(L3,1)+DP
C         START TIME LOOP
   19 DO 3  J=1,MN
      JJ=J+K4
      JJJ=J+K5
      AJ = J+KL2+1
      JA=AJ
      J3=JM+J
      J5=JN+J
      J1=JK+J
      J2=JL+J
C         REFLECTION FROM FREE LIQUID SURFACE
      DPR(1,JJ)=-DPL(2,J1)
C         FRICTION ORFICE ROUTINE FOR TANK RESISTANCE
      C=CT
      BF=BF1
      IF(LL-2) 92,93,93
   93 DO 6 L=2,LLL
  112 FORMAT(5E16.4)
    6 CALL FOR(PL(L,J),DPR(L-1,J1),DPL(L+1,J1),PR(L,J),V(L,J),PL (L,J+1)
     1,DPR (L,JJ),DPL (L,JJ),PR (L,J+1),V (L,J+1))
   92 VT(J) = VAA*SIN(CON*AJ)
C          TANK ROUTINE
      L=L1
      C1=CT
      C2=CF
      R=R1
      CALL TANK(VI(1,J),VO(1,J),PL(L,J),PR(L,J),DPR(L-1,J1),DPL(L+1,J2)
     1,VI(1,J+1),VO(1,J+1),PL(L,J+1),PR(L,J+1),DPL(L,JJ),DPR(L,JJ),VT(J))
     2)
C         FRICTION ORFICE ROUTINE FOR FEEDLING RESISTANCE
```

```
      C=CF
      BF=BF2
      DO 7L=LL2,LL3
    7 CALL FOR(PL(L,J),DPR(L-1,J2),DPL(L+1,J2),PR(L,J),V(L,J),PL (L,J+1)
     1,DPR (L,JJ),DPL (L,JJ),PR (L,J+1),V (L,J+1))
C          PUMP ROUTINE
      VA(J) = VBB*SIN(CON*AJ)
      L=L2
      R=R2
      C1=CF
      C=CC
      C2=CD
      CALL PUMP(VA(J),VI(2,J),VO(2,J),PL(L,J),PR(L,J),DPR(L-1,J2),DPL(L+
     11,J3),VI(2,J+1),VO(2,J+1),PL(L,J+1),PR(L,J+1),DPL(L,JJ), DPR(L,JJ)
     2)
C         FRICTION ORFICE ROUTINE FOR DISCHARGE LINE RESISTANCE
      C=CD
      BF=BF3
      DO 8 L=LL4,LL5
    8 CALL FOR(PL(L,J),DPR(L-1,J3),DPL(L+1,J3),PR(L,J),V(L,J),PL (L,J+1)
     1,DPR (L,JJ),DPL (L,JJ),PR (L,J+1),V (L,J+1))
C          INJECTOR ROUTINE
      L=L3
      CALL INJECT(PL(L,J),PR(L,J),V(L,J),QQ(L,J ),DPR(L-1,J3),PL(L,J+1),
     1PR(L,J+1),V(L,J+1),QQ(L,JJJ),DPL(L,JJ))
C         WRITE SOLUTION
      Y10=PL(L3,J+1)-X1
      Y12=PR(L3,J+1)-X12
      Y13=PR(L5,J+1)-X13
      Y14=PR(L6,J+1)-X14
      Y15=PR(L7,J+1)-X15
      Y16=PR(L8,J+1)-X16
      JA=AJ
      Y1=PR(L1,J+1)-XX1
      Y2=PL(L1,J+1)-X2
      Y5=PL(L2,J+1)-X5
      Y6=PR(L2,J+1)-X6
      Y3=QQ(L3,J+1)/AD-X3
      Y7=VI(1,J+1)-X7
      Y9=V(L3,J+1)-X9
      YY2(JA)=Y12
      WRITE(6,108)JA,Y2,Y1,Y5,Y6,Y10,Y12,Y3,Y7,Y9,Y13,Y14,Y15
  108 FORMAT(I5,    6F9.3,3F9.4,5F8.3)
      IF(ERROR.GE.100.) GO TO 17
      IF (J-KKK) 3,10,1
C         END TIME LOOP
    3 CONTINUE
  110 FORMAT(10G13.4)
   10 KL2 =KL2+KKK
```

```
C          RE-INIALIZATION
      DO 15  L=1,L3
      V(L,1) = V(L,KK2)
      DO 65 J=1,K5
      KKK1=KKK+J
   65 QQ(L3,J)=QQ(L3,KKK1)
      DO 15  J=1,K4
      KK = KKK+J
      DPL (L,J) = DPL(L,KK)
   15 DPR(L,J) = DPR(L,KK)
      DO 16 L=1,L3
      PL(L,1) = PL(L,KK2)
   16 PR (L,1) = PR(L,KK2)
      VI(1,1) = VI(1,KK2)
      VI(2,1) = VI(2,KK2)
      VO(1,1) = VO(1,KK2)
      VO(2,1) = VO(2,KK2)
      MN = MN-KKK
      IF(MN) 17,17,19
   17 IF(KODE2.NE.0) GO TO 2222
      GO TO 1
      END
```

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